Mr. Ward Answer Key

Rational Expressions

A rational expression is a polynomial divided by another polynomial.

Examples: $\frac{1}{3}, \frac{x+4}{5}, \frac{3x-7}{3x^5+6}$

Simplifying Rational Expressions

Step 1: Completely factor the Numerator & Denominator

- Common Monomial
- 3 Terms – Reverse Foil
- 2 Terms – Patterns
- 4 Terms – Grouping

Step 2: Divide out any common factors.

Step 3: Check for any values that would make the denominator zero. Get rid of them! (Remember we can't divide by zero! It's bad!)

Example #1 $\frac{x^2-5x}{x^2-2x-15}$

Let's go through each of the steps together.

Step 1: When I look at the top I see that both terms have a GCF of x. Therefore you should factor out an x from both terms in the top. In the bottom I see that I have 3 terms. Therefore I'm going to do reverse foil to factor it down. Do this below.

$\frac{x(x-5)}{(x-5)(x+3)}$

(Answer: $\frac{x(x-5)}{(x-5)(x+3)}$)

Step 2: Now that I have everything factored out, is there anything that both the top and bottom have in common? What is it? $(x-5)$ Let's cancel this from both the top and bottom then. Write below what we have left.

$\frac{x}{x+3}$

(Answer: $\frac{x}{x+3}$)

Step 3: The last thing we need to check for is that there aren't any values that make the denominator zero. To do this go back to before Step 2 when we had everything factored out. ($\frac{x(x-5)}{(x-5)(x+3)}$) Are there any values that make the denominator zero? ________

$x=5$ and $x=-3$

So our final answer would look like this: $\boxed{\frac{x}{x+3} \text{ where } x \neq -3 \text{ and } x \neq 5.}$

(This is our answer because the expression is as simplified as possible.)

<u>Practice Problem #1</u> $\frac{x^2-2x-3}{x^2+5x+4}$

Step 1: Factor the Numerator & Denominator

$$\frac{(x-3)(x+1)}{(x+4)(x+1)}$$

Step 2: Divide out any common factors

$$\frac{(x-3)}{(x+4)}$$

Step 3: Check for any values that would make the denominator zero.

$x=-4$ and $x=-1$

Final Answer = $\frac{x-3}{x+4}$ where $x \neq -4$ and $x \neq -1$

<u>Come show me your answer so I know you're on the right track!</u>

<u>Multiplying & Dividing Rational Expressions</u>

The steps are very similar to simplifying rational expressions so make sure you are paying close attention!

<u>Step 1:</u> Completely factor the Numerator & Denominator

<u>Step 2:</u> Divide out any common factors

<u>Step 3:</u> Multiply straight across **<u>OR</u>** when dividing, multiply by the reciprocal of the 2nd fraction.

<u>Step 4:</u> Check for any values that would make the denominator zero.

Example #1 $\frac{10x^3y^2}{7xy} \bullet \frac{3xy^2}{2x}$

Step 1 and 2: Look at the left fraction. Start by simplifying the x-values. There are 3 x's in the top and 1 in the bottom. Therefore after canceling we will have 2 remaining in the top. Do the same thing with the y-values. Now simplify the right fraction as well. Write your simplified expression below.

$$\frac{10x^2y}{7} \cdot \frac{3y^2}{2}$$

(Answer: $\frac{10x^2y}{7} \bullet \frac{3y^2}{2}$)

Step 3: Multiply the two fractions straight across. Multiply the top of the first fraction and the top of the second fraction together. Do the same things with the denominators. Write your answer below.

$$\frac{30x^2y^3}{14}$$

(Answer: $\frac{30x^2y^3}{14}$)

Step 4: There are no values that would make the denominator zero. Therefore all we have to do is simplify the expression to get our final answer.

Final Answer: $\frac{15x^2y^3}{7}$

Practice Problem #1 $\frac{8x^4y^2}{2x^3y} \div \frac{4x^2y}{7y}$

Step 1 and 2: Simplify the expression.

$$\frac{4xy}{1} \div \frac{4x^2}{7}$$

Step 3: Because this problem involves division you have to first flip the 2nd fraction to get the reciprocal. Then you can multiply straight across.

$$\frac{4xy}{1} \cdot \frac{7}{4x^2} = \frac{28xy}{4x^2} = \frac{7y}{x}$$

Step 4: Check for any values that make the denominator zero. If nothing, write the final answer.

$x = 0$

Final Answer: $\frac{7y}{x}$ where $x \neq 0$

Come show me your answer so I know you're on the right track!

Assignment: Problems Below + pg 580 #8-14

Simplify. Identify any *x*-values for which the expression is undefined.

1. $\dfrac{x^2+3x+2}{x^2-3x-4}$ $\qquad \dfrac{(x+2)\cancel{(x+1)}}{(x-4)\cancel{(x+1)}}$

$\dfrac{x+2}{x-4}$ where $x \neq 4, -1$

2. $\dfrac{4x^6}{2x^4}$

$2x^2$ where $x \neq 0$

3. $\dfrac{x^2-x^3}{2x^2-5x+3}$ $\qquad \dfrac{x^2(1-x)}{\cancel{(x-3)(2x+1)}\ (2x-3)(x-1)}$

where $x \neq 1, 3/2$

4. $\dfrac{x^3+x^2-20x}{x^2-16}$ $\qquad \dfrac{x(x^2+x-20)}{x^2-16}$ $\qquad \dfrac{x(x+5)\cancel{(x-4)}}{(x+4)\cancel{(x-4)}}$ $\qquad \dfrac{x(x+5)}{x+4}$

where $x \neq \pm 4$

5. $\dfrac{3x^2-9x-12}{6x^2+9x+3}$ $\qquad \dfrac{\cancel{3}(x^2-3x-4)}{\cancel{3}(2x^2+3x+1)}$ $\qquad \dfrac{(x-4)\cancel{(x+1)}}{(2x+1)\cancel{(x+1)}}$ $\qquad \dfrac{x-4}{2x+1}$

where $x \neq -1, -1/2$

6. $\dfrac{9-3x}{15-2x-x^2}$ $\qquad \dfrac{3(3-x)}{-1(x^2+2x-15)}$ $\qquad \dfrac{3(3-x)}{-1(x+5)(x-3)}$ $\qquad \dfrac{3}{x+5}$ where $x \neq -5, 3$

Multiply. Assume all expressions are defined.

7. $\dfrac{4x+16}{2x+6} \cdot \dfrac{x^2+2x-3}{x+4}$ $\qquad \dfrac{4\cancel{(x+4)}}{2\cancel{(x+3)}} \cdot \dfrac{\cancel{(x+3)}(x-1)}{\cancel{(x+4)}}$

$2(x-1)$

8. $\dfrac{x+3}{x-1} \cdot \dfrac{x^2-2x+1}{x^2+5x+6}$ $\qquad \dfrac{\cancel{x+3}}{\cancel{x-1}} \cdot \dfrac{\cancel{(x-1)}(x-1)}{\cancel{(x+3)}(x+2)}$

$\dfrac{x-1}{x+2}$

Divide. Assume all expressions are defined.

9. $\dfrac{5x^6}{x^2y} \div \dfrac{10x^2}{y}$ $\qquad \dfrac{5x^4}{\cancel{y}} \cdot \dfrac{\cancel{y}}{10x^2} = \dfrac{5x^4}{10x^2}$

$= \dfrac{x^2}{2}$

10. $\dfrac{x^2-2x-8}{x^2-2x-15} \div \dfrac{2x^2-8x}{2x^2-10x}$ $\qquad \dfrac{\cancel{(x-4)}(x+2)}{\cancel{(x-5)}(x+3)} \cdot \dfrac{\cancel{2x}\cancel{(x-5)}}{\cancel{2x}\cancel{(x-4)}}$

$\dfrac{x+2}{x+3}$

Solve. Check your solution.

11. $\dfrac{x^2+x-12}{x-3} = 15$ $\qquad \dfrac{(x+4)\cancel{(x-3)}}{\cancel{x-3}} = 15$

$x+4 = 15$

$x = 11$

12. $\dfrac{2x^2+8x-10}{2x^2+14x+20} = 4$ $\qquad \dfrac{\cancel{2}(x^2+4x-5)}{\cancel{2}(x^2+7x+10)} = 4$ $\qquad \dfrac{\cancel{(x+5)}(x-1)}{\cancel{(x+5)}(x+2)} = 4$

$\cancel{x+2}\left(\dfrac{x-1}{\cancel{x+2}}\right) = 4(x+2)$

$x - 1 = 4x + 8$

$-4x \quad -4x$

$-3x = 9$

$x = -3$

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8. $$\frac{x-2}{2x-3} \cdot \frac{4x-6}{x^2-4}$$

$$\frac{\cancel{x-2}}{\cancel{2x-3}} \cdot \frac{2(\cancel{2x-3})}{(x+2)(\cancel{x-2})}$$

$$= \boxed{\frac{2}{x+2}}$$

9. $$\frac{x-2}{x-3} \cdot \frac{2x-6}{x+5}$$

$$\frac{x-2}{\cancel{x-3}} \cdot \frac{2(\cancel{x-3})}{x+5}$$

$$= \boxed{\frac{2(x-2)}{x+5}}$$

10. $$\frac{x^2-16}{x^2-4x+4} \cdot \frac{x-2}{x^2+6x+8}$$

$$\frac{(\cancel{x+4})(x-4)}{(x-2)(\cancel{x-2})} \cdot \frac{\cancel{x-2}}{(\cancel{x+4})(x+2)}$$

$$= \boxed{\frac{x-4}{(x-2)(x+2)}}$$

11. $$\frac{x^5y^4}{3xy} \div \frac{1}{x^3y}$$

$$\frac{x^4y^3}{3} \div \frac{1}{x^3y} \xrightarrow{\text{flip}} \frac{x^4y^3}{3} \cdot \frac{x^3y}{1}$$

$$= \boxed{\frac{x^7y^4}{3}}$$

12. $$\frac{x+3}{x^2-2x+1} \div \frac{x+3}{x-1}$$

$$\frac{x+3}{(x-1)(x-1)} \div \frac{x+3}{x-1}$$

↓ flip

$$\frac{\cancel{x+3}}{(x-1)(\cancel{x-1})} \cdot \frac{\cancel{x-1}}{\cancel{x+3}}$$

$$= \boxed{\frac{1}{x-1}}$$

13. $$\frac{x^2-25}{2x^2+5x-12} \div \frac{x^2-3x-10}{x^2+9x+20}$$

$$\frac{(x+5)(x-5)}{(2x-3)(x+4)} \div \frac{(x-5)(x+2)}{(x+5)(x+4)}$$

↓ flip

$$\frac{(x+5)(\cancel{x-5})}{(2x-3)(\cancel{x+4})} \cdot \frac{(x+5)(\cancel{x+4})}{(\cancel{x-5})(x+2)}$$

$$= \boxed{\frac{(x+5)(x+5)}{(2x-3)(x+2)}}$$

14. $$\frac{x^2+2x+1}{x^2-3x-18} \div \frac{x^2-1}{x^2-7x+6}$$

$$\frac{(x+1)(x+1)}{(x-6)(x+3)} \div \frac{(x+1)(\cancel{x-1})}{(x-6)(\cancel{x-1})}$$

↓ flip

$$\frac{(x+1)(\cancel{x+1})}{(\cancel{x-6})(x+3)} \cdot \frac{\cancel{x-6}}{\cancel{x+1}}$$

$$= \boxed{\frac{x+1}{x+3}}$$