Mr. Ward Answer Key

Arithmetic Sequences and Series

A sequence or series is Arithmetic if all of its successive terms have the same *common difference.*

Example #1

$-3 + 2 + 7 + 12 + 17 + \ldots$
$\underbrace{\quad +5 \quad +5 \quad +5 \quad +5 \quad}$

What is the common difference between each of the terms? +5
If it helps, draw arrows between each of the terms.

(Answer: 5)

Practice Problem #1

Is the following sequence arithmetic? If so, find the common difference.

$-4, -12, -24, -40, -60, \ldots$
$\underbrace{\quad -8 \quad -12 \quad -16 \quad -20 \quad}$

Is the sequence arithmetic? NO Why? the successive terms of the sequence do not have the same common difference

Finding the n^{th} Term of an Arithmetic Sequence

What if I asked you to find the 100^{th} term of a sequence? That would take forever and you would not like it. Thankfully for you we have a general formula that will give us any n^{th} term we want.

$\boxed{a_n = a_1 + (n-1)d}$ where a_1 is the first term and d is the common difference.

Example #1

Find the 11^{th} term of the arithmetic sequence $-3, -5, -7, -9, \ldots$

Step 1: What is the common difference? -2

Step 2: Plug what you know into the formula and solve.

$$a_{11} = -3 + (11-1)\,-2$$
$$= -3 + (10) - 2$$
$$= -3 - 20$$
$$= -23$$

Answer = -23

Finding the Sum of an Arithmetic Series

Similar to finding a specific term of a sequence, you will also be asked to find the sum of series up to a certain point. This would also become very difficult if you were asked to find the sum of many consecutive numbers.

Just as before we will have a general formula that will help us solve these types of problems.

$\boxed{S_n = \frac{n}{2}(a_1 + a_n)}$ where a_1 and a_n are the first and last terms.

Example #1

Find S_{16} for the arithmetic series $12 + 7 + 2 + (-3) + \ldots$

Step 1: Find the common difference.

We can't use our formula quite yet because we don't know what the final term is. In order to find this we have to use the first formula we learned about to find a_{16}.

To start we need to know what the common difference is. Answer = -5

Step 2: Find the last term of the series.

Use the common difference and the formula $a_n = a_1 + (n-1)d$ to find the last term of the series. (16)

What is the last term? -63

$a_{16} = 12 + (16-1)-5$

$a_{16} = 12 - 75$

Step 3: Plug what you know into the formula and solve.

$a_{16} =$

Answer = -408

$S_{16} = \frac{16}{2}(12 + (-63))$

$= 8(-51) \quad = -408$

Practice Problem #1

Find the indicated sum for the arithmetic series $\sum_{k=1}^{12}(3+4k)$

Even though it doesn't look like it, this one is actually a little bit simpler. In order to find the sum we need to find both a_1 and a_{12}. Once we have those we can find the sum.

Use the formula [illegible] to find both answers.

What is a_1? 7

What is a_{12}? 51

Now plug all the parts you know into the formula to find the sum.

S_{12} = 348 $\qquad S_{12} = \frac{12}{2}(7+51) = 6(58) = 348$

Come show your answer to me so I know you're on the right track!!

Finding Missing Terms

Example #1 Find the missing terms in the arithmetic sequence 11, 4, -3, -10, -17

$a_n = a_1 + (n-1)d$

$-17 = 11 + (5-1)d$

$-28 = 4d$

$d = -7$

Finding n^{th} Term Given Two Terms

Example #1 Find the 6th term of the arithmetic sequence with $a_9 = 120$ and $a_{14} = 195$.

$a_n = a_1 + (n-1)d$

$a_{14} = a_9 + (14-9)d$

$195 = 120 + 5d$

$75 = 5d$

$15 = d$

$a_6 = a_9 + (n-9)d$

$a_6 = 120 + (6-9)15$

$= 120 + -45$

$a_6 = 75$

You should now be able to say the following:

- I can recognize whether a series (or sequence) is arithmetic (**C1**)
- I can use the formula to find the n^{th} term of an arithmetic sequence (**C2**)
- I can use the formula to find the sum of the first "n" terms of an arithmetic series (**C3**)

Assignment: **Practice B Worksheet + pg 884 #1-20**

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pg 884

1. arithmetic series

2. arithmetic ; $d = -7$; 11

3. not arithmetic

4. arithmetic ; $d = -2/3$; $2/3$

5. $a_8 = a_1 + (8-1)d$
$a_8 = 3 + (7)5$
$a_8 = 38$

6. $a_8 = a_1 + (8-1)d$
$a_8 = 10 + (7)-1/4$
$a_8 = 8.25$

7. $a_8 = a_1 + (8-1)d$
$a_8 = -3.2 + (7)-0.2$
$a_8 = -4.6$

8. $a_4 = a_1 + (4-1)d$
$25 = 13 + 3d$
$d = 4$
17, 21

9. $a_5 = a_1 + (5-1)d$
$37 = 9 + 4d$
$d = 7$
16, 23, 30

10. $a_5 = a_1 + (5-1)d$
$-1 = 1.4 + 4d$
$d = -0.6$
0.8, 0.2, -0.4

11. $a_5 = a_4 + (5-4)d$
$19 = 27 + d$
$d = -8$

12. $a_4 = a_3 + (4-3)d$
$12.6 = 12.2 + d$
$d = 0.4$

13. $a_6 = a_3 + (6-3)d$
$-11 = -5 + 3d$
$d = -2$

$a_9 = a_5 + (9-5)-8$
$a_9 = 19 + -32$
$a_9 = -13$

$a_9 = a_4 + (9-4)0.4$
$a_9 = 12.6 + 2$
$a_9 = 14.6$

$a_9 = a_6 + (9-6)-2$
$a_9 = -11 + -6$
$a_9 = -17$

14. $a_{20} = a_{10} + (20-10)d$
$50 = 100 + 10d$
$d = -5$

15. $a_{11} = a_7 + (11-7)d$
$-28 = -42 + 4d$
$d = 3.5$

16. $a_8 = a_4 + (8-4)d$
$1/2 = 3/4 + 4d$
$d = -1/16$

$a_9 = 105$

$a_9 = a_7 + (9-7)3.5$
$a_9 = -42 + 7$
$a_9 = -35$

$a_9 = 7/16$

→

17. $a_{15} = a_1 + (15-1)d$

$a_{15} = 5 + (14)4$

$a_{15} = 61$

$S_{15} = \frac{15}{2}(5 + 61)$

$\boxed{S_{15} = 495}$

18. $a_1 = 4$

$a_{12} = 70$

$S_{12} = \frac{12}{2}(70 + 4)$

$\boxed{S_{12} = 444}$

19. $a_{18} = a_1 + (18-1)d$

$a_{18} = 3.2 + (17)-0.3$

$a_{18} = -1.9$

$S_{18} = \frac{18}{2}(3.2 + -1.9)$

$\boxed{S_{18} = 11.7}$

20. a) $a_6 = a_1 + (6-1)d$

$a_6 = 26,000 + (5)1,250$

$\boxed{a_6 = 32,250}$

b) $S_6 = \frac{6}{2}(26,000 + 32,250)$

$\boxed{S_6 = 174,750}$