

LESSON
11-6 **Problem Solving**
Binomial Distributions

Sales records for the snack machines show that 1 out of every 6 students buys a bag of trail mix. There are 5 students waiting to use the machines. Melanie uses the formula for binomial probability, $P(r) = {}_n C_r p^r q^{n-r}$, to determine the number of students expected to buy trail mix.

- What is the probability of exactly 3 students buying a bag of trail mix?
 - What is the probability of each student buying a bag of trail mix? $\frac{1}{6}$
 - Define each variable used in the formula and give its value.

 - Write the binomial formula using these values. $P(3) = {}_5 C_3 \cdot (\frac{1}{6})^3 \cdot (\frac{5}{6})^2$
 $= 10 \cdot (\frac{1}{6})^3 \cdot (\frac{5}{6})^2$
 - Solve the equation to give the probability of exactly 3 students buying a bag of trail mix. $= 0.0322$ or 3.22%
- What is the probability of at least 1 student buying a bag of trail mix?
 - Describe a method to solve involving the sum of probabilities.
 $1 - P(0)$
 - Describe a method to solve that uses the formula $P(E) + P(\text{not } E) = 1$.
 $P(E) = 1 - P(\text{not } E)$
 - Use either method to determine the probability of at least 1 student buying a bag of trail mix.
 $P(0) = {}_5 C_0 \cdot (\frac{1}{6})^0 \cdot (\frac{5}{6})^5$
 $= 1 \cdot 1 \cdot (\frac{5}{6})^5$
 $= 0.4019 \rightarrow 1 - 0.4019 = 0.5981$
- After school, 4 students line up to buy snacks from the machine. What is the probability that they will all buy a bag of trail mix?
 $P(4) = {}_4 C_4 \cdot (\frac{1}{6})^4 \cdot (\frac{5}{6})^0 = 0.0008$

Sports drinks are purchased by 3 out of 4 students using the snack machines. There are 3 students at the machines now. Choose the letter for the best answer.

- Which expression gives the probability of exactly 2 students buying an energy drink?
 - $P(2) = {}_3 C_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^1$
 - $P(2) = {}_4 C_3 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^1$
 - $P(3) = {}_3 C_2 \left(\frac{3}{4}\right)^1 \left(\frac{1}{4}\right)^2$
 - $P(3) = {}_4 C_3 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^2$
- What is the probability that at least 2 of the students will buy an energy drink?
 - 42%
 - 50%
 - 75%
 - 84%

$P(2) + P(3)$
 $= {}_3 C_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^1 + {}_3 C_3 \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)^0$
 $= 3 \cdot \left(\frac{3}{4}\right)^2 \cdot \left(\frac{1}{4}\right) + 1 \cdot \left(\frac{3}{4}\right)^3 \cdot 1$
 $= 0.8438$

KEEP

SCROLLING

DOWN

Practice B

$$\begin{aligned} 5. a) P(3) &= {}_4C_3 \cdot \left(\frac{1}{6}\right)^3 \cdot \left(\frac{5}{6}\right)^1 \\ &= 4 \cdot \left(\frac{1}{6}\right)^3 \cdot \left(\frac{5}{6}\right) \\ &= 0.0154 \text{ or } 1.54\% \end{aligned}$$

$$\begin{aligned} b) \text{ At least } 2 &\rightarrow P(2) + P(3) + P(4) \\ &= {}_4C_2 \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^2 + 0.0154 + {}_4C_4 \cdot \left(\frac{1}{6}\right)^4 \cdot \left(\frac{5}{6}\right)^0 \\ &= 6 \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^2 + 0.0154 + 1 \cdot \left(\frac{1}{6}\right)^4 \cdot 1 \\ &= 0.1157 + 0.0154 + 0.0008 \\ &= 0.1319 \text{ or } 13.19\% \end{aligned}$$

$$\begin{aligned} 6. a) P(\text{all } 5) &= {}_5C_5 \cdot \left(\frac{1}{3}\right)^5 \cdot \left(\frac{2}{3}\right)^0 \\ &= 1 \cdot \left(\frac{1}{3}\right)^5 \cdot 1 \\ &= 0.0041 \text{ or } 0.41\% \end{aligned}$$

$$\begin{aligned} b) P(1) &= {}_5C_1 \cdot \left(\frac{1}{3}\right)^1 \cdot \left(\frac{2}{3}\right)^4 \\ &= 5 \cdot \frac{1}{3} \cdot \left(\frac{2}{3}\right)^4 \\ &= 0.3292 \text{ or } 32.92\% \end{aligned}$$

$$\begin{aligned} 7. P(4) &= {}_8C_4 \cdot \left(\frac{1}{3}\right)^4 \cdot \left(\frac{2}{3}\right)^4 \\ &= 70 \cdot \left(\frac{1}{3}\right)^4 \cdot \left(\frac{2}{3}\right)^4 \\ &= 0.1767 \text{ or } 17.07\% \end{aligned}$$

$$\begin{aligned} 8. \text{ At least } 1 &\rightarrow P(1) + P(2) + \dots + P(10) \\ \text{Instead } &\rightarrow 1 - P(0) \end{aligned}$$

$$\begin{aligned} P(0) &= {}_{10}C_0 \cdot \left(\frac{1}{10}\right)^0 \cdot \left(\frac{9}{10}\right)^{10} \\ &= 1 \cdot 1 \cdot \left(\frac{9}{10}\right)^{10} \\ &= 0.3487 \end{aligned}$$

$$1 - 0.3487 = 0.6513 \text{ or } 65.13\%$$

Practice C

$$\begin{aligned} 5. a) P(2) &= {}_4C_2 \cdot \left(\frac{1}{7}\right)^2 \cdot \left(\frac{6}{7}\right)^2 \\ &= 6 \cdot \left(\frac{1}{7}\right)^2 \cdot \left(\frac{6}{7}\right)^2 \\ &= 0.09 \quad \text{or} \quad 9\% \end{aligned}$$

$$\begin{aligned} b) \text{ at least } 3 &\rightarrow P(3) + P(4) \\ &= {}_4C_3 \cdot \left(\frac{1}{7}\right)^3 \cdot \left(\frac{6}{7}\right)^1 + {}_4C_4 \cdot \left(\frac{1}{7}\right)^4 \cdot \left(\frac{6}{7}\right)^0 \\ &= 4 \cdot \left(\frac{1}{7}\right)^3 \cdot \left(\frac{6}{7}\right) + 1 \cdot \left(\frac{1}{7}\right)^4 \cdot 1 \\ &= 0.009996 \quad + \quad 0.00042 \\ &= 0.0104 \quad \text{or} \quad 1.04\% \end{aligned}$$

$$\begin{aligned} 6. a) P(5) &= {}_5C_5 \cdot \left(\frac{2}{5}\right)^5 \cdot \left(\frac{3}{5}\right)^0 \\ &= 1 \cdot \left(\frac{2}{5}\right)^5 \cdot 1 \\ &= 0.0102 \quad \text{or} \quad 1.02\% \end{aligned}$$

$$\begin{aligned} b) P(0) &= {}_5C_0 \cdot \left(\frac{2}{5}\right)^0 \cdot \left(\frac{3}{5}\right)^5 \\ &= 1 \cdot 1 \cdot \left(\frac{3}{5}\right)^5 \\ &= 0.0778 \quad \text{or} \quad 7.78\% \end{aligned}$$

$$\begin{aligned} 7. P(3) &= {}_8C_3 \cdot \left(\frac{1}{5}\right)^3 \cdot \left(\frac{4}{5}\right)^5 \\ &= 56 \cdot \left(\frac{1}{5}\right)^3 \cdot \left(\frac{4}{5}\right)^5 \\ &= 0.1468 \quad \text{or} \quad 14.68\% \end{aligned}$$

$$8. \text{ at least } 3 \rightarrow P(3) + P(4) + P(5) + P(6)$$

$$\begin{aligned} &= {}_6C_3 \cdot \left(\frac{1}{12}\right)^3 \cdot \left(\frac{11}{12}\right)^3 + {}_6C_4 \cdot \left(\frac{1}{12}\right)^4 \cdot \left(\frac{11}{12}\right)^2 + {}_6C_5 \cdot \left(\frac{1}{12}\right)^5 \cdot \left(\frac{11}{12}\right)^1 + {}_6C_6 \cdot \left(\frac{1}{12}\right)^6 \cdot \left(\frac{11}{12}\right)^0 \\ &= 20 \cdot \left(\frac{1}{12}\right)^3 \cdot \left(\frac{11}{12}\right)^3 + 15 \cdot \left(\frac{1}{12}\right)^4 \cdot \left(\frac{11}{12}\right)^2 + 6 \cdot \left(\frac{1}{12}\right)^5 \cdot \left(\frac{11}{12}\right)^1 + 1 \cdot \left(\frac{1}{12}\right)^6 \cdot 1 \\ &= 0.008915 \quad + \quad 0.000608 \quad + \quad 0.000022 \quad + \quad 0.0000003 \\ &= 0.009545 \quad \text{or} \quad 0.95\% \end{aligned}$$