

Exponential Functions: Growth and Decay

The first thing we need to do before we begin is look at the formula for an exponential function. What does it look like?

$f(x) = ab^x$ where "a" is the starting value, "b" is the base and "x" is the exponent.

Put a big circle or box around that equation. **IT IS VERY IMPORTANT!!!**

Tables of Values:

Let's look at a very standard exponential function: $y = 3^x$

If we were to create a table with domain and range values it would look like this:

| x | y |
|---|----|
| 0 | 1 |
| 1 | 3 |
| 2 | 9 |
| 3 | 27 |
| 4 | 81 |

Something very important happens with this table of values, specifically the y-values. Fill in the blanks below to see what this is.

$$\frac{3}{1} = \frac{3}{3} \quad \frac{9}{3} = \frac{3}{9} \quad \frac{27}{9} = \frac{3}{27} \quad \frac{81}{27} = \frac{3}{81}$$

What did you notice? they are all 3

Notice how that 3 comes up in our original equation? Look back at it if you don't. This will happen with **all** exponential functions. They will either increase or decrease by a consistent rate.

Practice Problem: Given the following table, write an exponential function.

| x | y |
|---|-----|
| 0 | 1 |
| 1 | 1/2 |
| 2 | 1/4 |
| 3 | 1/8 |

$$\frac{1/2}{1} = \frac{1/2}{1/2} \quad \frac{1/4}{1/2} = \frac{1/2}{1/4} \quad \frac{1/8}{1/4} = \frac{1/2}{1/8}$$

$$y = \left(\frac{1}{2}\right)^x$$

Answer: $y = \left(\frac{1}{2}\right)^x$

Sometimes we will have exponential growth and other times exponential decay.

Read on to the next page to learn the difference between these two.

Exponential Growth:

When you see the graph of an exponential growth function, the graph will rise as the x-values increase. (Exactly what you would expect.) As far as our parent function goes, "b" (which stood for the base) will be greater than one ($b > 1$).

Examples: $y = 2^x$, $y = 1.5^x$, $y = 5(4)^x$

Notice with that 3rd example the starting value (the "a") is 5 instead of 1.

Exponential Decay:

When we have the graph of an exponential decay function, the graph will fall as the x-values increase. (Again, exactly what you would expect.) When we look at the parent function now, "b" will be between 0 and 1 ($0 < b < 1$).

Examples: $y = .5^x$, $y = \left(\frac{1}{3}\right)^x$, $y = 2\left(\frac{3}{4}\right)^x$

Again with that 3rd example, the starting value is 2. This would affect what the graph of this function looks like.

Applications:

One of the ways we use exponential functions in everyday life and problems is modeled through the following equation: $A(t) = a(1 \pm r)^t$

"A(t)" = final amount

"a" = initial amount

"r" = rate

"t" = time

When the equation is $(1+r)$ we will have exponential growth.

When the equation is $(1-r)$ we will have exponential decay.

Practice Problem: You put \$2500 in the bank at 4% annual interest. How much will you have after 50 years?

$$\begin{aligned} A(t) &= 2500 (1 + 0.04)^{50} \\ &= 2500 (1.04)^{50} \end{aligned}$$

$$A(t) = 17,766.71$$

Answer: \$17,766.71

Assignment: Worksheet + pg 493 #2-10, 15, 17, 21

LESSON
7-1

Practice B
Exponential Functions, Growth, and Decay

Tell whether the function shows growth or decay. Then graph.

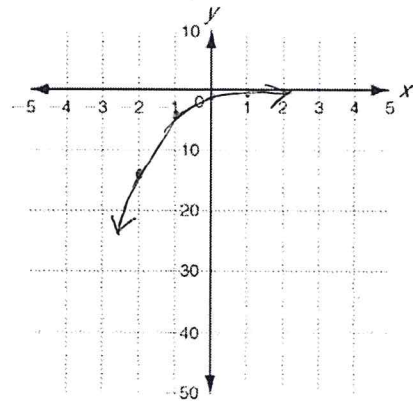
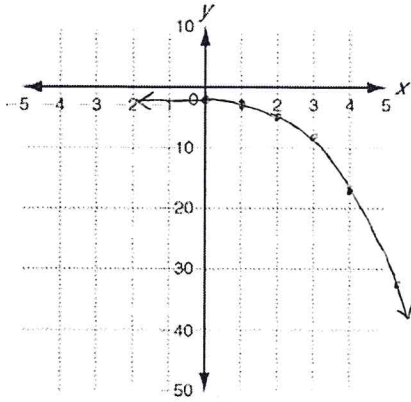
1. $g(x) = -(2)^x$

2. $h(x) = -0.5(0.2)^x$

Growth

Decay

| X | Y |
|---|-----|
| 0 | -1 |
| 1 | -2 |
| 2 | -4 |
| 3 | -8 |
| 4 | -16 |
| 5 | -32 |



| X | Y |
|----|-------|
| -2 | -12.5 |
| -1 | -2.5 |
| 0 | -0.5 |
| 1 | -0.1 |

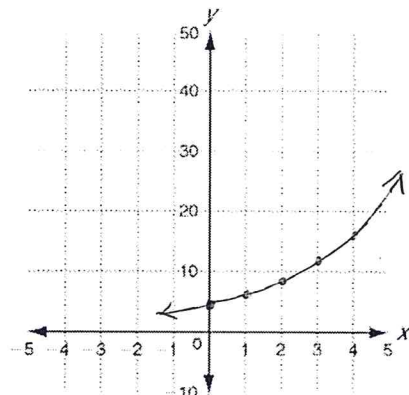
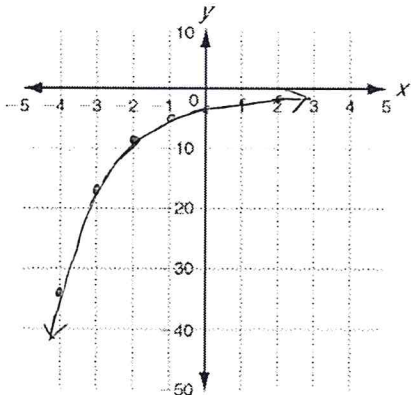
3. $j(x) = -2(0.5)^x$

4. $p(x) = 4(1.4)^x$

Decay

Growth

| X | Y |
|----|-------|
| 0 | -2 |
| 1 | -1 |
| 2 | -0.5 |
| 3 | -0.25 |
| -1 | -4 |
| -2 | -8 |
| -3 | -16 |
| -4 | -32 |



| X | Y |
|---|-------|
| 0 | 4 |
| 1 | 5.6 |
| 2 | 7.84 |
| 3 | 10.98 |
| 4 | 15.37 |

Solve.

5. A certain car depreciates about 15% each year.

a. Write a function to model the depreciation in value for a car valued at \$20,000.

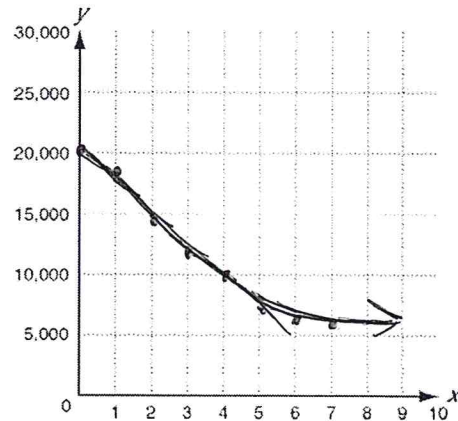
$A(t) = 20,000(1 - 0.15)^t$

b. Graph the function.

c. Suppose the car was worth \$20,000 in 2005. What is the first year that the value of this car will be worth less than half of that value?

the 5th year

| X | Y |
|---|--------|
| 0 | 20,000 |
| 1 | 17,000 |
| 2 | 14,450 |
| 3 | 12,282 |
| 4 | 10,440 |
| 5 | 8,874 |
| 6 | 7,542 |



Mr. Ward Homework Key

pg 493

2. Exponential Decay
See Graph

3. Exponential Growth
See Graph

4. Exponential Decay
See Graph

5. a) $f(x) = 150(2)^x$
b) See Graph
c) 614,400

6. a) $f(x) = 25\left(\frac{2}{5}\right)^x$
b) See Graph
c) After 4 bounces

7. Exponential Decay
See Graph

8. Exponential Growth
See Graph

9. Exponential Growth
See Graph

10. a) $A(t) = 580(1 + 0.0232)^t$
 $A(t) = 580(1.0232)^t$

b) See Graph

c) Year 24 ($t=24$)

15. 2008
-1626

382 years

$A(t) = 24(1 + 0.035)^{382}$

$A(t) = 24(1.035)^{382}$

$A(t) = 12,229,955.10$

17. $A(t) = 2765(1 - 0.3)^x$
 $350 = 2765(1 - 0.3)^x$
 $\frac{350}{2765} = \frac{2765}{2765}(0.7)^x$

$0.12658 = (0.7)^x$

$x \approx 5.8$ years

21. a) $A(t) = 12,000(1 - 0.2)^6$

$A(t) = 12,000(0.8)^6$

$A(t) = 3146$ beanbag animals

b) $A(t) = 12,000(0.8)^x$

$1000 = 12,000(0.8)^x$

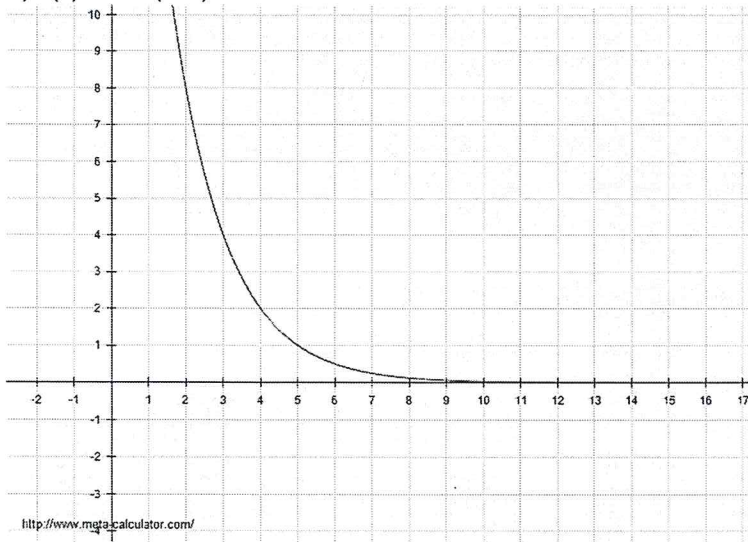
$0.08\bar{3} = (0.8)^x$

$x \approx 12^{\text{th}}$ month (11.23)

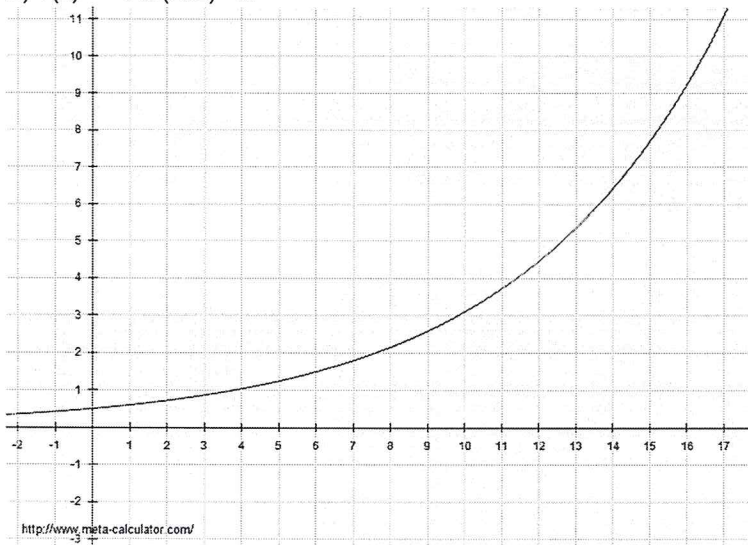
7-1 Graphs

Pg 493 # 2-10

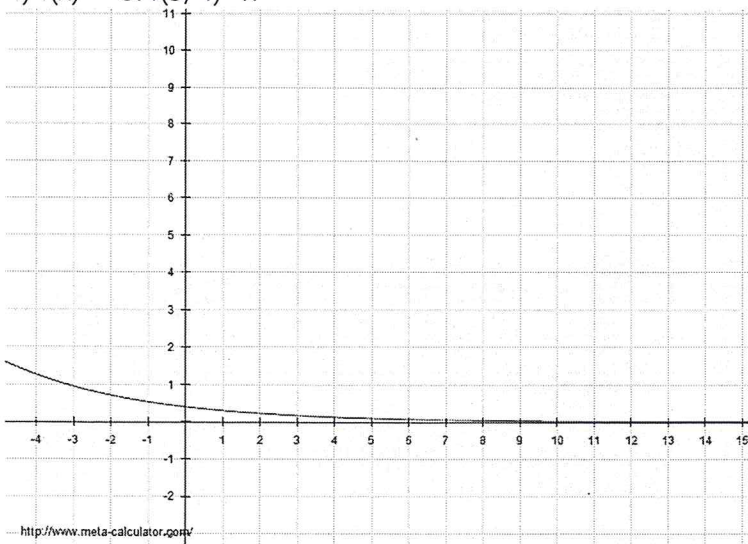
2) $f(x) = 32(0.5)^x$



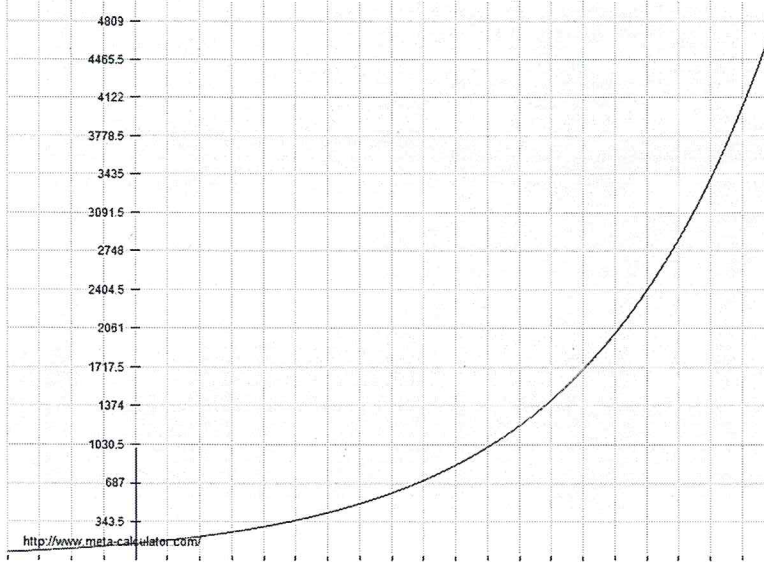
3) $f(x) = 0.5(1.2)^x$



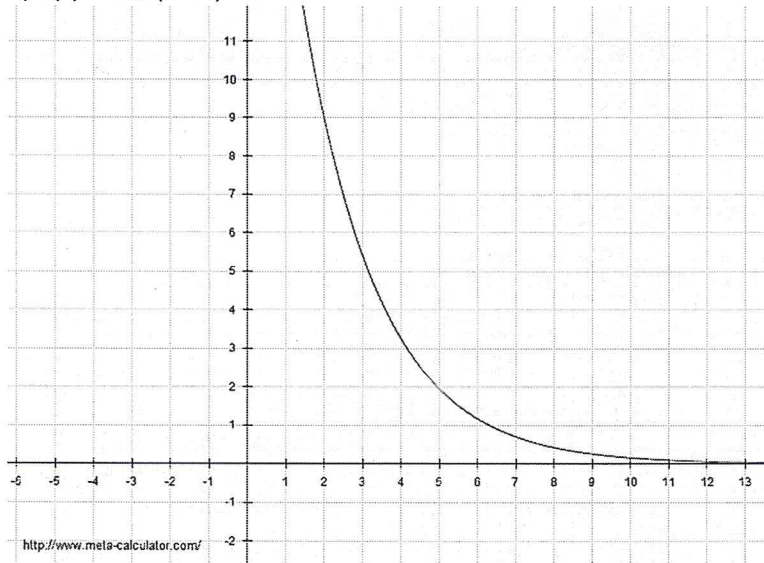
4) $f(x) = 0.4(3/4)^x$



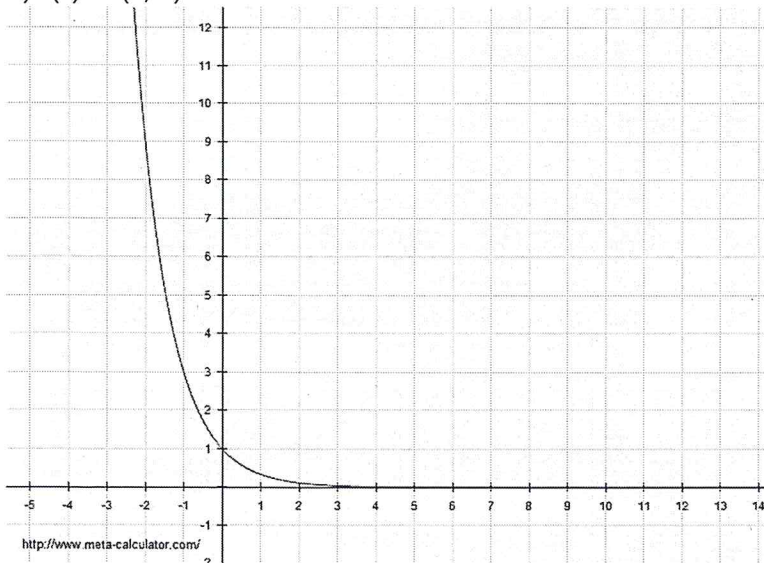
5) $f(x) = 150(2)^x$



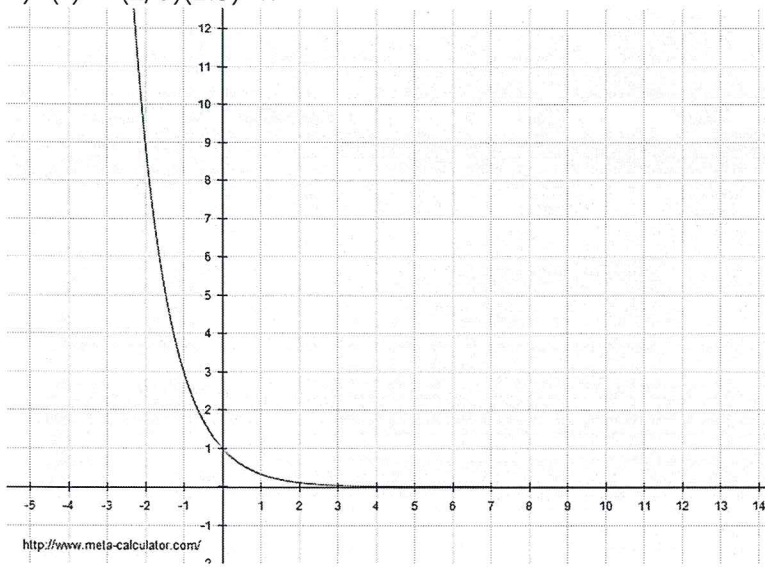
6) $f(x) = 25(1-.4)^x$



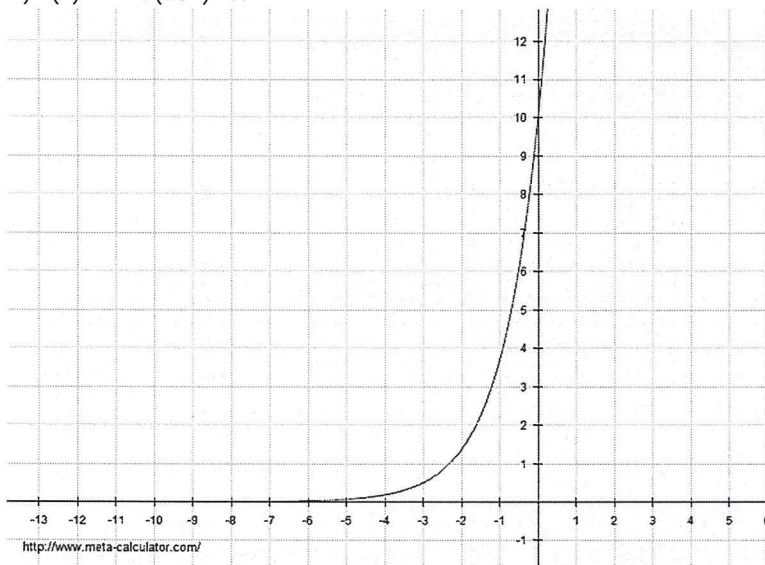
7) $f(x) = (1/3)^x$



8) $f(x) = (1/3)(1.3)^x$



9) $f(x) = 10(2.7)^x$



10) $f(x) = 580(1.0232)^x$

