Mr. Ward Answer Key

Finding Roots

Remember all the factoring you've done over the past few days? It's important for what we're about to do next. We are going to use that factoring to find roots of polynomials.

Roots are the solutions or x-intercepts of polynomials. They are the values of the polynomial that when substituted in make the polynomial equal zero.

To find these roots or solutions, once a polynomial is factored (something you already know how to do), you set each factor equal to zero. You then solve for x. What you get for x is your root.

Example: $2y^3 + 4y^2 - 30y$

Factor the polynomial. DON'T do any solving yet, JUST factor. Do this below.

$2y(y^2 + 2y - 15)$

$2y(y+5)(y-3)$

You should have gotten to here: $2y(y+5)(y-3)$

Now we are just going to take each of these factors and set them equal to zero.

$2y = 0$ $\quad$ $y+5=0$ $\quad$ $y-3=0$

$\quad\quad\quad\quad$ $-5 \; -5$ $\quad$ $+3 \; +3$

$\boxed{y=0}$ $\quad$ $\boxed{y=-5}$ $\quad$ $\boxed{y=3}$

Solve each equation for y. The three answers you get for y are your solutions/roots.

Do Problems: See Zeros Worksheet

So let's step back and think about how we've solved things the past couple days.

We've gone from POLYNOMIAL → FACTORS → ROOTS/SOLUTIONS/ZEROS

Couldn't we just go the other way if we wanted to? The answer is YES!

ROOTS/SOLUTIONS/ZEROS → FACTORS → POLYNOMIAL

Example: Write the simplest polynomial function with zeros -1, 1, 3.

The first thing you need to do is take these zeros and put them into factors. Write these three factors below. They should all be linear.

$(x+1)$, $(x-1)$, and $(x-3)$.

Now take these 3 factors and multiply them together. Start with just 2. Then multiply the 3rd factor after the first 2. Show your work below.

$(x^2-1)(x-3)$

$\boxed{= x^3 - 3x^2 - x + 3}$

SORRY! NO SPACE

After you're done, check that you've gotten $x^3 - 3x^2 - x + 3$. If not, ask for help!

Do Problems:

Write the simplest polynomial function given the following zeros.

1. 3, 2, -2

$(x-3)(x-2)(x+2)$

$x^2 - 5x + 6\ (x+2)$

$x^3 + 2x^2 - 5x^2 - 10x + 6x + 12$

$\boxed{= x^3 - 3x^2 - 4x + 12}$

4. -3, ½, 5

$(x+3)(x-1/2)(x-5)$

$x^2 - 2x - 15\ (x - 1/2)$

$x^3 - \frac{1}{2}x^2 - 2x^2 + x - 15x + 7\frac{1}{2}$

$\boxed{= x^3 - 2\frac{1}{2}x^2 - 14x + 7\frac{1}{2}}$

2. 3, 1, -2, -4

$(x-3)(x-1)(x+2)(x+4)$

$(x^2 - 4x + 3)(x^2 + 6x + 8)$

	x^2	$6x$	8
x^2	x^4	$6x^3$	$8x^2$
$-4x$	$-4x^3$	$-24x^2$	$-32x$
3	$3x^2$	$18x$	24

$\boxed{= x^4 + 2x^3 - 13x^2 - 14x + 24}$

5. 4, 1, -1

$(x-4)(x-1)(x+1)$

$x^2 - 5x + 4\ (x+1)$

$x^3 + x^2 - 5x^2 - 5x + 4x + 4$

$\boxed{= x^3 - 4x^2 - x + 4}$

3. 5, -1, 0, 2

$(x-5)(x+1)(x)(x-2)$

$x(x^2 - 4x - 5)(x-2)$

$x(x^3 - 2x^2 - 4x^2 + 8x - 5x + 10)$

$x(x^3 - 6x^2 + 3x + 10)$

$\boxed{= x^4 - 6x^3 + 3x^2 + 10x}$

6. 2, 3, -5

$(x-2)(x-3)(x+5)$

$x^2 - 5x + 6\ (x+5)$

$x^3 + 5x^2 - 5x^2 - 25x + 6x + 30$

$\boxed{= x^3 - 19x + 30}$

Quadratic Formula

Another way to find roots of a polynomial is through using the quadratic formula. However we can only use the quadratic formula when we have a trinomial.

How many terms in a trinomial again? 3

We typically use the quadratic formula when we have a trinomial that we can't factor *easily*. The formula for the quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Do you remember where the a, b, and c come from? Remember standard form?

When a polynomial is in standard form ($ax^2 + bx + c = 0$), the coefficients correspond to the a, b, and c of the quadratic formula. <u>You must have your polynomial set equal to zero before you do the quadratic formula.</u>

Solve each polynomial using the Quadratic Formula.

1. $4x^2 + 4x - 9 = 0$

$a=4 \quad b=4 \quad c=-9$

$$\frac{-4 \pm \sqrt{16-4(4)(-9)}}{2(4)}$$

$$\frac{-4 \pm \sqrt{160}}{8} = \frac{-4 \pm 4\sqrt{10}}{8}$$

$$= -\frac{1}{2} \pm \frac{\sqrt{10}}{2}$$

2. $9x^2 - 11 = 6x$

$9x^2 - 6x - 11 = 0$

$a=9 \quad b=-6 \quad c=-11$

$$\frac{6 \pm \sqrt{36-4(9)(-11)}}{2(9)}$$

$$\frac{6 \pm \sqrt{432}}{18} = \frac{6 \pm 12\sqrt{3}}{18}$$

$$= \frac{1}{3} \pm \frac{2\sqrt{3}}{3}$$

3. $8n^2 - 4n = 18$

$8n^2 - 4n - 18 = 0$

$a=8 \quad b=-4 \quad c=-18$

$$\frac{4 \pm \sqrt{16-4(8)(-18)}}{2(8)}$$

$$\frac{4 \pm \sqrt{592}}{16} = \frac{4 \pm 4\sqrt{37}}{16}$$

$$= \frac{1}{4} \pm \frac{\sqrt{37}}{4}$$

4. $2n^2 - n - 4 = 2$

$2n^2 - n - 6 = 0$

$a=2 \quad b=-1 \quad c=-6$

$$\frac{1 \pm \sqrt{1-4(2)(-6)}}{2(2)}$$

$$\frac{1 \pm \sqrt{49}}{4} = \frac{1 \pm 7}{4} = 2, -3/2$$

5. $2a^2 - a - 13 = 2$

$2a^2 - a - 15 = 0$

$a=2 \quad b=-1 \quad c=-15$

$$\frac{1 \pm \sqrt{1-4(2)(-15)}}{2(2)}$$

$$\frac{1 \pm \sqrt{121}}{4} = \frac{1 \pm 11}{4} = 3, -2.5$$

Zeros Worksheet

Name Mr. Ward

Find all zeros. | Then multiply together to get the original polynomial.

1. $f(x) = (2x-1)(x-5)$

$2x-1=0$ $\quad$ $x-5=0$

$+1 \quad +1$ $\quad$ $+5 \quad +5$

$2x=1$ $\quad$ $\boxed{x=5}$

$\boxed{x=1/2}$

$2x^2-10x-x+5$

$\boxed{=2x^2-11x+5}$

2. $f(x) = (x-3)(3x+1)(x+1)$

$\boxed{x=3}$ $\quad$ $3x+1=0$ $\quad$ $\boxed{x=-1}$

$-1 \quad -1$

$3x=-1$

$\boxed{x=-1/3}$

$3x^2+x-9x-3\ (x+1)$

$3x^2-8x-3\ (x+1)$

$3x^3+3x^2-8x^2-8x-3x-3$

$\boxed{=3x^3-5x^2-11x-3}$

3. $f(x) = (2x+1)(x+1)(x-1)$

$\boxed{x=-1/2}$ $\quad$ $\boxed{x=-1}$ $\quad$ $\boxed{x=1}$

$2x^2+2x+x+1\ (x-1)$

$2x^2+3x+1\ (x-1)$

$2x^3-2x^2+3x^2-3x+x-1$

$\boxed{=2x^3+x^2-2x-1}$

4. $f(x) = x(x+2)(x-2)(3x^2-4)$

$\boxed{x=0}$ $\quad$ $\boxed{x=-2}$ $\quad$ $\boxed{x=2}$

$3x^2-4=0$

$3x^2=4$

$\sqrt{x^2}=\sqrt{\frac{4}{3}}$

$x=\pm\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$

$\boxed{x=\pm\frac{2\sqrt{3}}{3}}$

$x(x^2-4)(3x^2-4)$

$x(3x^4-4x^2-12x^2+16)$

$x(3x^4-16x^2+16)$

$\boxed{=3x^5-16x^3+16x}$

5. $f(x) = x(2x-1)(x+2)(x-2)$

$\boxed{x=0}$ $\quad$ $\boxed{x=1/2}$ $\quad$ $\boxed{x=-2}$ $\quad$ $\boxed{x=2}$

$x(2x-1)(x^2-4)$

$x(2x^3-8x-x^2+4)$

$\boxed{=2x^4-x^3-8x^2+4x}$

(Put in Standard Form)

6. $f(x) = (3x-2)(x+1)$

$3x-2=0$ $\quad$ $\boxed{x=-1}$

$3x=2$

$\boxed{x=2/3}$

$=3x^2+3x-2x-2$

$\boxed{=3x^2+x-2}$