Mr. Ward Answer Key

Geometric Sequences and Series

A sequence or series is Geometric if all of its successive terms have the same *common* *ratio*.

Example #1

128, 64, 32, 16, ...

1/2 1/2 1/2

What is the common ratio between each of the terms? 1/2

If it helps, draw arrows between each of the terms.

(Answer: ½)

Practice Problem #1

Is the following sequence arithmetic or geometric? If possible, find the common ratio or difference.

8, 12, 18, 27

In order to be 100% sure whether the sequence is arithmetic or geometric you should find both the difference between each term and the ratio between each term.

Differences: +4, +6, +9

Ratios: 1.5, 1.5, 1.5

Arithmetic or Geometric? Geometric Why? all of the successive terms have the same common ratio

Finding the n^th^ term of a Geometric Sequence

What if I asked you to find the 100th term of a sequence? That would take forever and you would not like it. Thankfully for you we have a general formula that will give us any nth term we want.

$\boxed{a_n = a_1 \cdot r^{n-1}}$ where a_1 is the first term and r is the common ratio.

Example #2

Find the 9th term of the arithmetic sequence –5, 10, –20, 40, ...

Step 1: What is the common ratio? -2

Step 2: Plug what you know into the formula and solve.

$a_n = -5 \cdot (-2)^{9-1}$

$= -5 \cdot (-2)^8$

Answer = -1,280

$a_n = -1,280$

Practice Problem #2

Find the 10th term of the geometric sequence with $a_5 = 96$ and $a_7 = 384$.

(*Use the same formula, but now use what you know, i.e. what you were given.*)

$a_7 = a_5 \cdot r^{7-5}$

$\frac{384}{96} = \frac{96r^2}{96}$

$4 = r^2$

$\boxed{r = \pm 2}$

$a_{10} = a_5 \cdot r^{10-5}$ $\quad$ $a_{10} = a_5 \cdot r^{10-5}$

$a_{10} = 96(2)^5$ $\quad$ $a_{10} = 96(-2)^5$

$\boxed{a_{10} = 3,072 \quad \text{OR} \quad a_{10} = -3,072}$

Similar to finding a specific term of a sequence, you will also be asked to find the sum of series up to a certain point. This would also become very difficult if you were asked to find the sum of many consecutive numbers.

Just as before we will have a general formula that will help us solve these types of problems.

$$\boxed{S_n = a_1 \cdot \left(\frac{1-r^n}{1-r}\right)}, \text{ where } r \neq 1$$

Example #3

Find S_8 for the geometric series 3 – 6 + 12 – 24 + ...

Step 1: Find the common ratio.

Common Ratio = -2

Step 2: Plug what you know into the formula and solve. (*Be careful with your negative signs!*) ~~Find a_8 first!~~

$\cancel{a_8 = 3 \cdot (-2)^{8-1}}$

$\cancel{a_8 = -384}$

Answer = -255

$S_8 = 3 \cdot \left(\frac{1-(-2)^8}{1-(-2)}\right)$

$= 3 \cdot \left(\frac{1-256}{3}\right) = -255$

Practice Problem #1

Find the indicated sum for the geometric series $\sum_{k=1}^{6} -3(2)^{k-1}$

Even though it doesn't look like it, this one is actually a little bit simpler. In order to find the sum we need to find two things, r and a_1. Once we have those we can find the sum.

To find a1 we need to plug 1 in for k.

What is a_1? ___-3___

Now to find r, all we have to do is look back at our original formula. The *common ratio* "r" will be the thing in parenthesis.

What is r? ___2___

Now plug all the parts you know into the formula to find the sum.

$S_6 = -3 \cdot \left(\frac{1-(2)^6}{1-(2)} \right) = -3 \cdot \left(\frac{1-64}{-1} \right)$

$= -3 \cdot 63$

$S_6 =$ ___-189___

Come show your answer to me so I know you're on the right track!!

You should now be able to say the following:

- I can recognize whether a sequence or series is geometric (**D1**)
- I can use the formula to find the nth term of a geometric sequence (**D2**)
- I can use the formula to calculate the sum of the first "n" terms of a geometric series (**D3**)

Assignment: ~~**Practice B Worksheet**~~ **+ pg 895 #2-10, 14-18**

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pg 895

2. Arithmetic ; $d = -2$ 3. Neither 4. Geometric ; $r = 1/4$

5. $a_{10} = 2 \cdot (3)^{10-1}$

$a_{10} = 39,366$

6. $a_{10} = 5000 \cdot (1/10)^{10-1}$

$a_{10} = 0.000005$

7. $a_{10} = -0.125 \cdot (-2)^{10-1}$

$a_{10} = 64$

8. $r = 1/3$

$\boxed{a_6 = -4/3}$

9. $a_5 = a_2 \cdot r^{5-2}$

$108 = 4 \cdot r^3$

$r = 3$

$a_6 = a_2 \cdot (3)^{6-2}$

$a_6 = 4 \cdot (3)^4$

$\boxed{a_6 = 324}$

10. $a_5 = a_3 \cdot r^{5-3}$

$12 = 3 \cdot r^2$

$r = \pm 2$

$a_6 = a_5 \cdot (2)^{6-5}$

$a_6 = 12 \cdot 2$

$\boxed{a_6 = \pm 24}$

14. $r = 1/10$

$S_6 = 2 \cdot \left(\frac{1 - (1/10)^6}{1 - (1/10)} \right)$

$S_6 = 2.22222$

15. $a_1 = 1$

$S_5 = 1 \cdot \left(\frac{1 - (-3)^5}{1 - (-3)} \right)$

$S_5 = 61$

16. $r = -2$

$S_5 = 12 \cdot \left(\frac{1 - (-2)^5}{1 - (-2)} \right)$

$S_5 = 132$

17. $a_1 = 256$

$S_9 = 256 \cdot \left(\frac{1 - (1/2)^9}{1 - (1/2)} \right)$

$S_9 = 511$

18. $r = 1.05$

$a_{20} = 32,000 \cdot (1.05)^{20-1}$

$\boxed{a_{20} = 80,862}$

$S_{20} = 32,000 \left(\frac{1 - (1.05)^{20}}{1 - (1.05)} \right)$

$\boxed{S_{20} = 1,058,110.53}$