

LESSON

**Reteach**

LESSON RETEACH

14-1

**Graphs of Sine and Cosine**

Transformations of the sine and cosine functions change the amplitude and/or the period of the graph.

For  $y = a\sin bx$  or  $y = a\cos bx$ :

- the amplitude is  $|a|$ ,
- the period is  $\frac{2\pi}{|b|}$ .

The amplitude is half the difference between the greatest and least values of the function.

One full cycle appears in each period.

Use the graph of  $f(x) = \sin x$  to sketch the graph of  $g(x) = 0.5\sin 2x$ .

**Step 1** Compare  $g(x) = 0.5\sin 2x$  to  $y = a\sin bx$ . Find  $a$  to identify the amplitude.

$a = 0.5$  and  $|0.5| = 0.5$ , so the amplitude is 0.5.

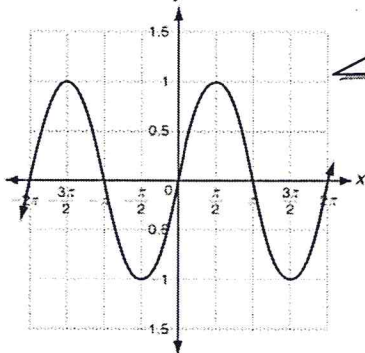
The maximum value of  $g(x)$  is 0.5 and the minimum value is  $-0.5$ .

**Step 2** Find  $b$  to identify the period.

$b = 2$ , and  $\frac{2\pi}{|b|} = \frac{2\pi}{|2|} = \pi$ , so the period is  $\pi$ .

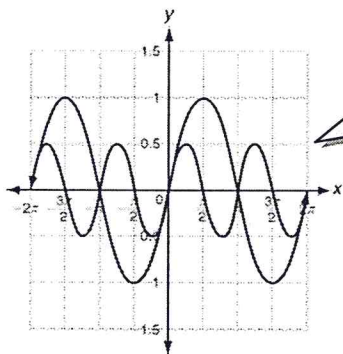
One full cycle appears in the interval from 0 to  $\pi$ .

**Step 3** Graph  $f(x) = \sin x$ .



The **amplitude** is 1. The maximum and minimum values of  $f(x)$  are 1 and  $-1$ . The **period** is  $2\pi$ . One full cycle appears in the interval from 0 to  $2\pi$ . Two full cycles appear in the interval from  $-2\pi$  to  $2\pi$ . The  $x$ -intercepts are at multiples of  $\pi$ .

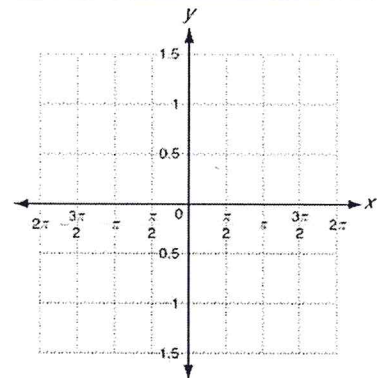
**Step 4** Graph  $g(x) = 0.5\sin 2x$  on the same plane as  $f(x)$ .



The **amplitude** is 0.5. The maximum and minimum values of  $g(x)$  are 0.5 and  $-0.5$ . The **period** is  $\pi$ . One full cycle appears in the interval from 0 to  $\pi$ . Two full cycles appear in the interval from 0 to  $2\pi$  and from  $-2\pi$  to 0. The  $x$ -intercepts are at multiples of  $\frac{\pi}{2}$ .

**Complete to graph  $h(x) = 0.5\cos 2x$ .**

1. Find the amplitude of  $h(x)$ .  $a =$  \_\_\_\_\_
2. Find the period of  $h(x)$ .  $\frac{2\pi}{|b|} =$  \_\_\_\_\_
3. What are the maximum and minimum values of  $h(x)$ ? \_\_\_\_\_
4. How many full cycles appear in the interval from 0 to  $\pi$ ? \_\_\_\_\_
5. Sketch the graph of  $f(x) = \cos x$ . Then graph  $h(x) = 0.5\cos 2x$ .



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**Graphs of Sine and Cosine (continued)**

A **phase shift** is a horizontal translation. Sine and cosine can be translated horizontally by  $y = \sin(x - h)$  and  $y = \cos(x - h)$ .

A phase shift (or horizontal translation) of  $h$  units moves the graph left  $h$  units for  $h < 0$  or right  $h$  units for  $h > 0$ .

Use the graph of  $f(x) = \cos x$  to sketch the graph of  $g(x) = \cos\left(x - \frac{\pi}{2}\right)$ .

**Step 1** Compare  $g(x) = \cos\left(x - \frac{\pi}{2}\right)$  to  $y = a \cos bx$ .

Find the amplitude and period.

$a = 1$  and  $|1| = 1$ , so the amplitude is 1.

$b = 1$ , and  $\frac{2\pi}{|b|} = \frac{2\pi}{|1|} = 2\pi$ , so the period is  $2\pi$ .

The amplitude and period of  $g$  are the same as for  $y = \cos x$ .

**Step 2** Find  $h$  to identify the phase shift.

$x - h = x - \frac{\pi}{2}$ , so  $h = \frac{\pi}{2}$ .

Because  $h > 0$ , the shift is to the right.

The phase shift is  $\frac{\pi}{2}$  radians to the right.

**Step 3** Identify the first two positive  $x$ -intercepts.

The  $x$ -intercepts of  $f(x) = \cos x$  occur at  $\frac{\pi}{2}$  and  $\frac{3\pi}{2}$ .

The  $x$ -intercepts of  $g(x)$  occur at  $\frac{\pi}{2} + \frac{\pi}{2}$ , or  $\pi$ , and  $\frac{3\pi}{2} + \frac{\pi}{2}$ , or  $2\pi$ .

Intercepts occur at integer multiples of  $\pi$ .

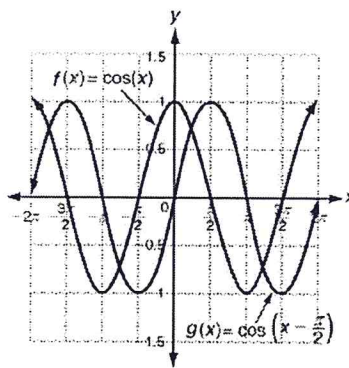
**Step 4** Identify the maximum and minimum values.

The maxima and minima of  $f(x) = \cos x$  occur at 0 and  $\pi$ .

The maxima and minima of  $g(x)$  occur at

$0 + \frac{\pi}{2}$ , or  $\frac{\pi}{2}$ , and  $\pi + \frac{\pi}{2}$ , or  $\frac{3\pi}{2}$ .

**Step 5** Graph  $f(x) = \cos x$  and  $g(x) = \cos\left(x - \frac{\pi}{2}\right)$ .



Use the graph of  $f(x) = \sin x$  to sketch the graph of  $g(x) = \sin\left(x - \frac{\pi}{2}\right)$ .

6. Identify  $h$ . What is the phase shift?

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7. Identify the  $x$ -intercepts from 0 to  $2\pi$ .

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8. Identify the maxima and minima from 0 to  $2\pi$ .

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9. Sketch the graphs of  $f(x)$  and  $g(x)$ .

