Name Mr. Ward Date _____ Class _____

LESSON 14-1

Reteach

Graphs of Sine and Cosine

Transformations of the sine and cosine functions change the amplitude and/or the period of the graph.

For $y = a\sin bx$ or $y = a\cos bx$:

- the amplitude is $|a|$,
- the period is $\frac{2\pi}{|b|}$.

The amplitude is half the difference between the greatest and least values of the function.

One full cycle appears in each period.

Use the graph of $f(x) = \sin x$ to sketch the graph of $g(x) = 0.5\sin 2x$.

Step 1 Compare $g(x) = 0.5\sin 2x$ to $y = a\sin bx$.
Find a to identify the amplitude.

$a = 0.5$ and $|0.5| = 0.5$, so the amplitude is 0.5.

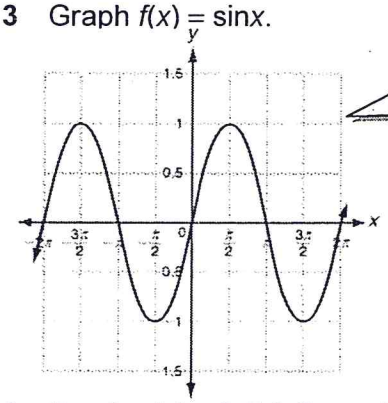
The maximum value of $g(x)$ is 0.5 and the minimum value is –0.5.

Step 2 Find b to identify the period.

$b = 2$, and $\frac{2\pi}{|b|} = \frac{2\pi}{|2|} = \pi$, so the period is π.

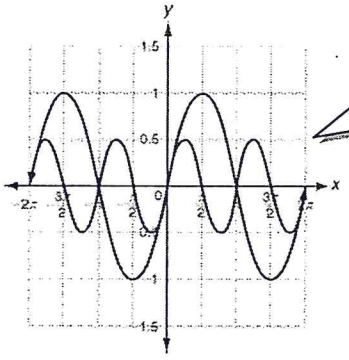
One full cycle appears in the interval from 0 to π.

Step 3 Graph $f(x) = \sin x$.



The **amplitude** is 1. The maximum and minimum values of $f(x)$ are 1 and –1. The **period** is 2π. One full cycle appears in the interval from 0 to 2π. Two full cycles appear in the interval from -2π to 2π. The x-intercepts are at multiples of π.

Step 4 Graph $g(x) = 0.5\sin 2x$ on the same plane as $f(x)$.

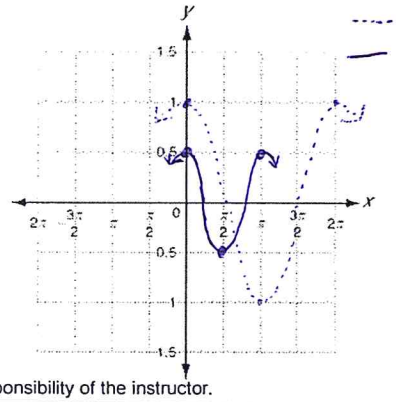


The **amplitude** is 0.5. The maximum and minimum values of $g(x)$ are 0.5 and –0.5. The **period** is π. One full cycle appears in the interval from 0 to π. Two full cycles appear in the interval from 0 to 2π and from -2π to 0.

The x-intercepts are at multiples of $\frac{\pi}{2}$.

Complete to graph $h(x) = 0.5\cos 2x$.

1. Find the amplitude of $h(x)$. $a =$ 0.5
2. Find the period of $h(x)$. $\frac{2\pi}{|b|} =$ $2\pi/2 = \pi$
3. What are the maximum and minimum values of $h(x)$? max = 0.5 min = -0.5
4. How many full cycles appear in the interval from 0 to π? 1 full cycle
5. Sketch the graph of $f(x) = \cos x$. Then graph $h(x) = 0.5\cos 2x$.



---- = f(x)
— = h(x)

Name Mr. Ward Date _____ Class _____

LESSON 14-1

Reteach

Graphs of Sine and Cosine *(continued)*

A **phase shift** is a horizontal translation. Sine and cosine can be translated horizontally by $y = \sin(x - h)$ and $y = \cos(x - h)$.
A phase shift (or horizontal translation) of h units moves the graph left h units for $h < 0$ or right h units for $h > 0$.

Use the graph of $f(x) = \cos x$ to sketch the graph of $g(x) = \cos\left(x - \frac{\pi}{2}\right)$.

Step 1 Compare $g(x) = \cos\left(x - \frac{\pi}{2}\right)$ to $y = \boldsymbol{a}\cos\boldsymbol{b}x$.

Find the amplitude and period.
$a = 1$ and $|1| = 1$, so the amplitude is 1.
$b = 1$, and $\frac{2\pi}{|b|} = \frac{2\pi}{|1|} = 2\pi$, so the period is 2π.

The amplitude and period of g are the same as for $y = \cos x$.

Step 2 Find h to identify the phase shift.

$x - h = x - \frac{\pi}{2}$, so $h = \frac{\pi}{2}$.

Because $h > 0$, the shift is to the right.

The phase shift is $\frac{\pi}{2}$ radians to the right.

Step 3 Identify the first two positive x-intercepts.

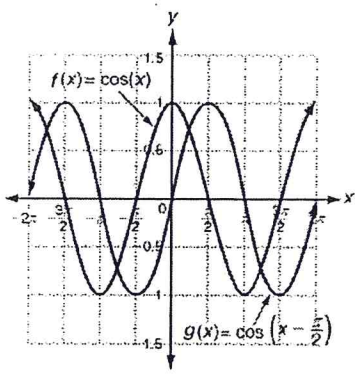
The x-intercepts of $f(x) = \cos x$ occur at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

The x-intercepts of $g(x)$ occur at $\frac{\pi}{2} + \frac{\pi}{2}$, or π, and $\frac{3\pi}{2} + \frac{\pi}{2}$, or 2π.

Intercepts occur at integer multiples of π.

Step 4 Identify the maximum and minimum values.
The maxima and minima of $f(x) = \cos x$ occur at 0 and π.
The maxima and minima of $g(x)$ occur at $0 + \frac{\pi}{2}$, or $\frac{\pi}{2}$, and $\pi + \frac{\pi}{2}$, or $\frac{3\pi}{2}$.

Step 5 Graph $f(x) = \cos x$ and $g(x) = \cos\left(x - \frac{\pi}{2}\right)$.



Use the graph of $f(x) = \sin x$ to sketch the graph of $g(x) = \sin\left(x - \frac{\pi}{2}\right)$.

---- : f(x)
—— : g(x)

6. Identify h. What is the phase shift?

π/2 to the right

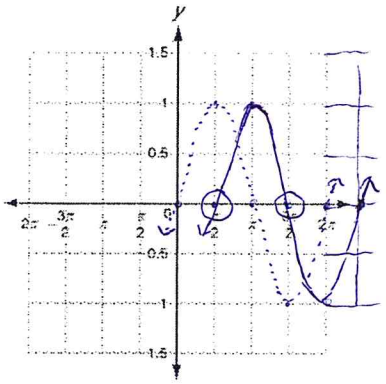
7. Identify the x-intercepts from 0 to 2π.

π/2 and 3π/2

8. Identify the maxima and minima from 0 to 2π.

max = 1 min = -1

9. Sketch the graphs of $f(x)$ and $g(x)$.



Pg 995 # 4-6, 8-10, 14-17, 19-22, 33-34

Mr. Ward Answer Key

4. $f(x) = 2 \sin \frac{1}{2} x$

amp = 2 period = $\frac{2\pi}{|1/2|} = 4\pi$

5. $h(x) = \frac{1}{4} \cos x$

amp = $1/4$ period = $\frac{2\pi}{|1|} = 2\pi$

6. $k(x) = \sin \pi x$

amp = 1 period = $\frac{2\pi}{|\pi|} = 2$

8. $f(x) = \sin\left(x + \frac{3\pi}{2}\right)$

phase shift = $3\pi/2$ to the left

x-intercepts = $-3\pi/2, -\pi/2, \pi/2$

9. $g(x) = \cos(x - \pi/2)$

phase shift = $\pi/2$ to the right

x-intercepts = $\pi, 2\pi$

10. $h(x) = \sin(x - \pi/4)$

phase shift = $\pi/4$ to the right

x-intercepts = $\pi/4, 5\pi/4, 9\pi/4$

14. $f(x) = 4 \cos x$

amp = 4 period = $\frac{2\pi}{|1|} = 2\pi$

15. $g(x) = \frac{3}{2} \sin x$

amp = $3/2$ period = $\frac{2\pi}{|1|} = 2\pi$

16. $g(x) = -\cos 4x$

amp = $|-1| = 1$ period = $\frac{2\pi}{|4|} = \pi/2$

17. $j(x) = 6 \sin \frac{1}{3} x$

amp = 6 period = $\frac{2\pi}{|1/3|} = 6\pi$

19. $f(x) = \sin(x + \pi)$

phase shift = π to the left

x-intercepts = $-\pi, 0, \pi$

20. $h(x) = \cos(x - 3\pi)$

phase shift = 3π to the right

x-intercepts =

21. $g(x) = \sin(x + 3\pi/4)$

phase shift = $3\pi/4$ to the left

x-intercepts =

22. $j(x) = \cos(x + \pi/4)$

phase shift = $\pi/4$ to the left

x-intercepts =

33. $f(x) = 6 \sin 2x$

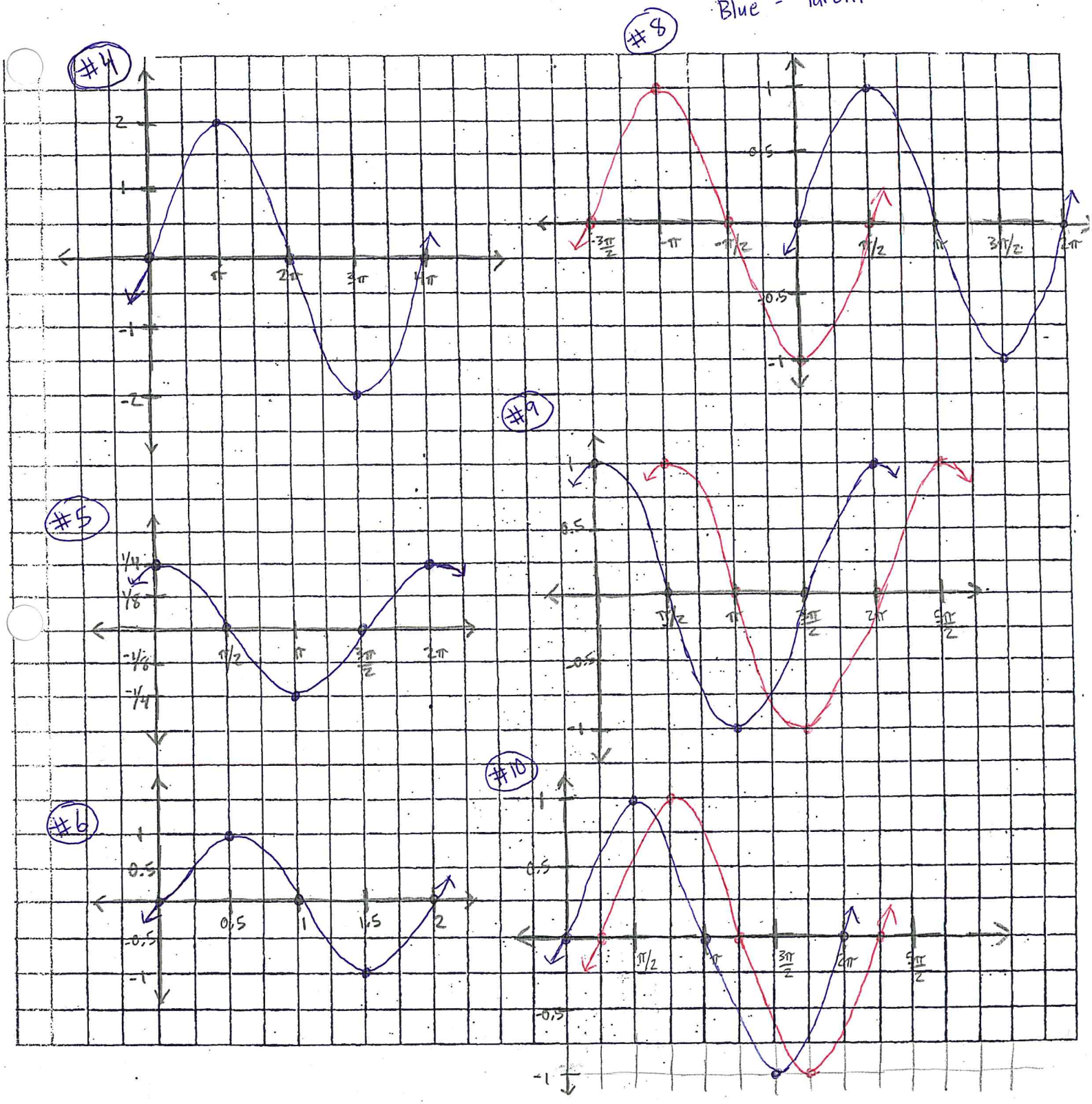
$f(x) = 6 \cos 2x$

34. $f(x) = \frac{1}{4} \sin(x + 2\pi/3)$

$f(x) = \frac{1}{4} \cos(x + 2\pi/3)$

Red = Final Answer

Blue = Parent

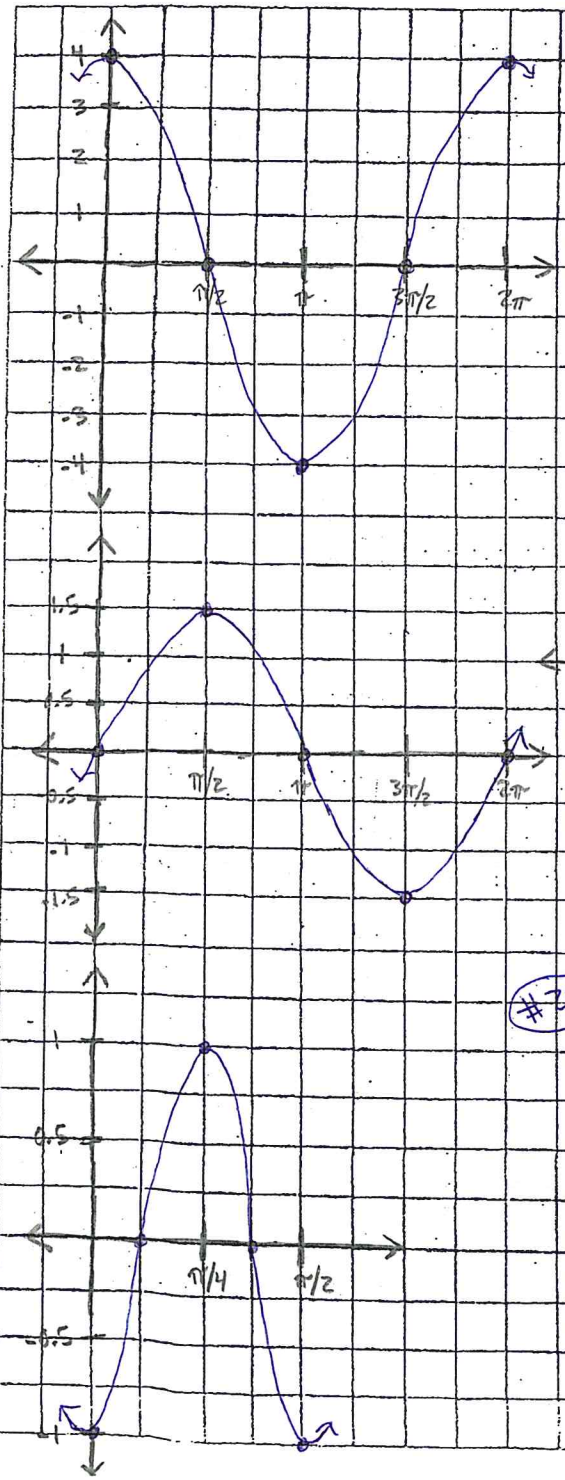


Blue = Parent

Red = Final Answer

5

#14



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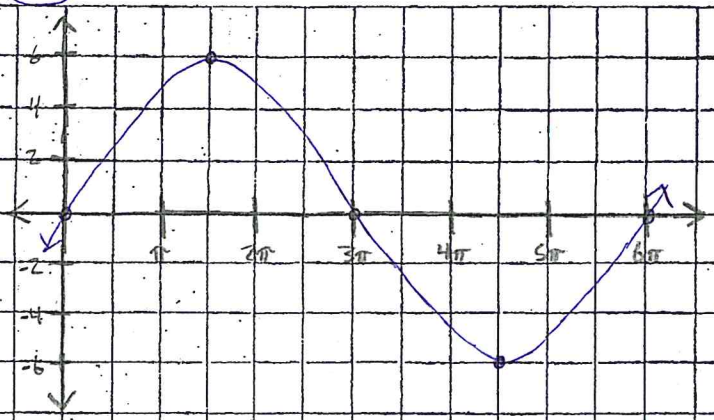

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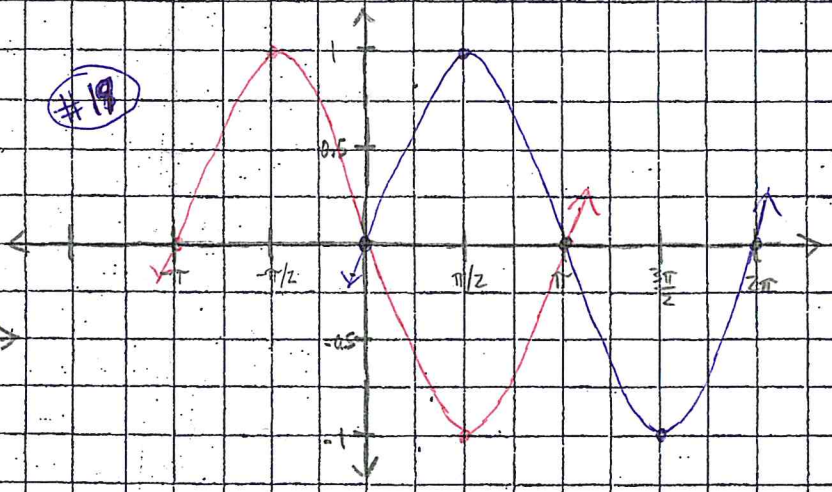

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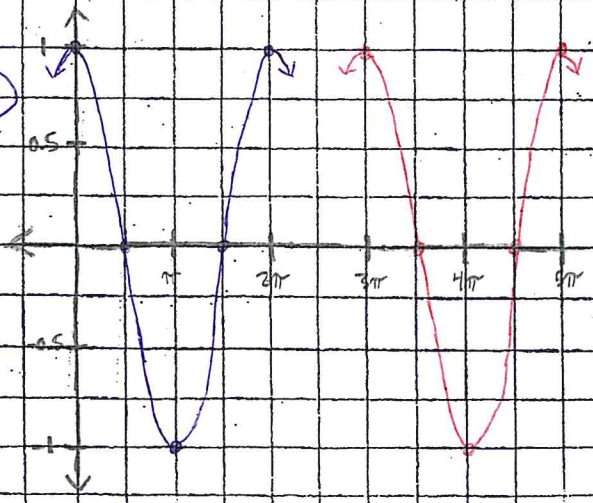

Blue = Parent

Red = Final Answer

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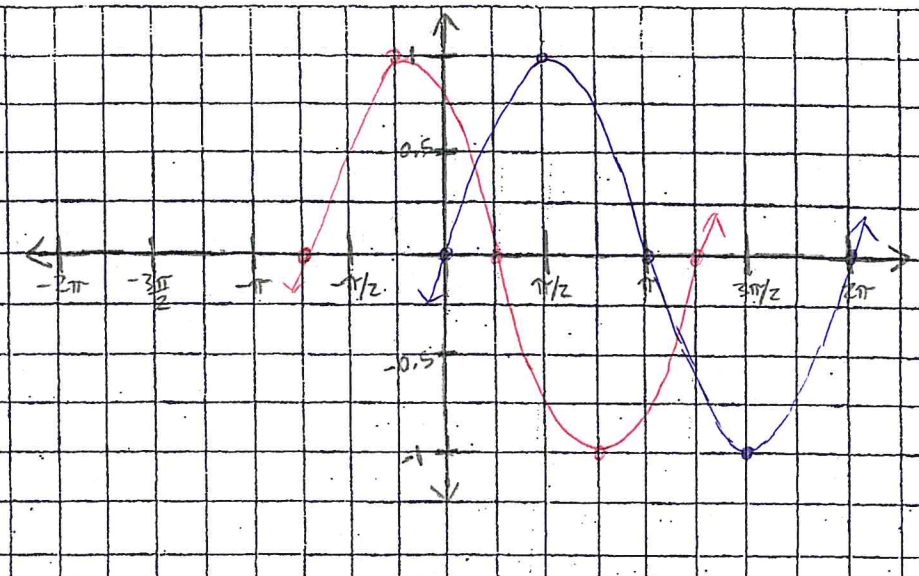

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