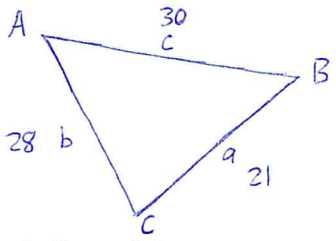
Algebra 2B-----Law of Cosines

*** Remember from yesterday that Law of Sines and Law of Cosines are used to solve triangles that are NOT right triangles. ***

1. Let's get started! Draw a triangle below that is NOT a right triangle:



2. Label the Angles A, B, and C.

3. Now put one finger on angle A. Good, now put another finger on the side that is opposite angle A (the side that doesn't touch A at all!!!). Label this side "a" (use a lower case letter this time.) Do the same thing to label side "b" and side "c".

4. In your picture above, label the following, side a = 21, side b = 28, and side c = 30. Put those numbers in your picture above.

5. Now, for the next minute or so I want you to try to use the Law of Sine's to find out how many degrees angle A is (it won't work, but it's important that you try so you can see why). Do your work in the space below:

We don't have angles to use in the equation!

$$\frac{\sin A}{21} = \frac{\sin B}{28}$$

6. When you've tried and failed, give a good explanation below why the Law of Sines will not help you solve for any of the angles:

NO ANGLES!!

7. So, we need a new formula, and here it is:

Law of Cosines: $\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$

8. In review, lower case letters always stand for sides of the triangle.

9. Upper case letters always stand for the angles of the triangle.

10. So, this new formula has 3 sides and 1 angle in the formula.

11. This is good, because in the problem we are trying to solve above, we know 3 sides and are trying to find out what the one angle is! Let's use our new equation.

12. Go back to problem #6 and tell me what angle we were trying to find. Angle A

13. Ok, you should have said Angle A. Now look at the formula. We are going to have to put that in for the capital A, because that's the only place we have an angle in there. I rewrote the formula below, but put a blank where I want you to fill in the A. Put the A in the formula:

$$a^2 = b^2 + c^2 - 2bc\cos\underline{A}$$

14. Remember that the sides of the triangle we have are side a = 21, side b = 28, and side c = 30. Remember that angles and sides have to correspond with each other. So if we have capital A, where should side a (21) go into the equation:

$$\underline{21}^2 = \_\_^2 + \_\_^2 - 2(\_\_)(\_\_)\cos A$$

15. Now put the 28 and the 30 into the formula. (hint they just have to go into the last blanks. It doesn't matter which one you make the b and which one you make the c).

$$21^2 = \underline{28}^2 + \underline{30}^2 - 2(\underline{28})(\underline{30})\cos A$$

16. Ok, this is an important step. I wrote below how you were supposed to fill all the values in. If you are confused about why the numbers were placed where they were, you should <u>definitely</u> call me over before you continue!

$$21^2 = 28^2 + 30^2 - 2(28)(30)\cos A$$

17. Ok, now it's time to solve. You have to get A by itself. There are lots of ways you can put this into your calculator and get it right, but on the next page I am going to suggest one way that I think is the easiest.

18. Solve for A in the following equation:

$$21^2 = 28^2 + 30^2 - 2(28)(30)\cos A$$

Step 1: Square 21, 28, and 30 in the equation above and rewrite you equation below:

$$\underline{441} = \underline{784} + \underline{900} - 2(28)(30)\cos A$$

Step 2: Add the 784 and the 900 together to get 1684. Then subtract this from the 441 on the right side:

$$\underline{-1243} = -2(28)(30)\cos A$$

Step 3: Multiply the -2, the 28, and the 30 together and rewrite:

$$-1243 = -1680 \cos A$$

Step 4: Divide each side by -1680 and rewrite:

$$0.7399 = \cos A$$

Step 5: It's inverse time!!! Finish the problem up and show your work/answer below:

$\cos^{-1}(0.7399) = \cos^{-1} \cos A \implies A = 42.3°$

19. Did you get 42.3 as your answer? If so, then you just solved a very difficult question and should pretty much feel like an awesome person. If not, try the steps in your calculator a few more times. If you still don't get it, call me over!

*** Finish the rest of these problems. As always with these problems check your answers as you are working through. ***

In ΔABC, find the measure of the missing sides and/or angles to the nearest tenth.

1. a = 13, b = 20, c = 9 Find m∠A.

$13^2 = 20^2 + 9^2 - 2(20)(9)\cos A$

$169 = 400 + 81 - 360 \cos A$

$-312 = -360 \cos A$

$\frac{-312}{-360} = \cos A$

$A = \cos^{-1}\left(\frac{-312}{-360}\right)$

$A = 29.9°$

2. a = 4, b = 8, m∠C = 56° Find c.

$c^2 = 4^2 + 8^2 - 2(4)(8)\cos 56°$

$c^2 = 16 + 64 - 64 \cos 56°$

$c^2 = 80 - 35.788$

$c^2 = 44.212$

$c = 6.6$

3. a = 6, b = 8, m∠C = 41° Find c.

$c^2 = 6^2 + 8^2 - 2(6)(8)\cos 41°$

$c^2 = 36 + 64 - 96 \cos 41°$

$c^2 = 100 - 72.452$

$c^2 = 27.548$

$c = 5.2$

4. a = 3, b = 7, c = 9 Find m∠B.

$7^2 = 3^2 + 9^2 - 2(3)(9)\cos B$

$49 = 9 + 81 - 54 \cos B$

$-41 = -54 \cos B$

$\frac{-41}{-54} = \cos B$

$\cos^{-1}\left(\frac{-41}{-54}\right) = \cos^{-1} \cos B$

$B = 40.6°$

Practice B WS

1. $m^2 = 8^2 + 14^2 - 2(8)(14)\cos 94°$

$m^2 = 64 + 196 - 224\cos 94°$

$m^2 = 260 - (-15.625)$

$m^2 = 275.625$

$\boxed{m = 16.6}$

$\frac{\sin 94}{16.6} = \frac{\sin N}{14}$

$\frac{14 \cdot \sin 94}{16.6} = \frac{16.6 \sin N}{16.6}$

$0.8413 = \sin N$

$\boxed{N = 57.3°}$

$180 - 94 - 57.3 = L$

$\boxed{L = 28.7°}$

2. $n^2 = 3^2 + 13^2 - 2(3)(13)\cos 36°$

$n^2 = 9 + 169 - 78\cos 36°$

$n^2 = 178 - 63.103$

$n^2 = 114.897$

$\boxed{n = 10.7}$

$\frac{\sin 36}{10.7} = \frac{\sin M}{3}$

$\frac{3 \cdot \sin 36}{10.7} = \frac{10.7 \sin M}{10.7}$

$0.165 = \sin M$

$\boxed{M = 9.5°}$

$180 - 36 - 9.5 = L$

$\boxed{L = 134.5°}$

3. $m^2 = 24^2 + 29^2 - 2(24)(29)\cos 65°$

$m^2 = 576 + 841 - 1392\cos 65°$

$m^2 = 1417 - 588.285$

$m^2 = 828.715$

$\boxed{m = 28.8}$

$\frac{\sin 65°}{28.8} = \frac{\sin N}{24}$

$\frac{24 \cdot \sin 65}{28.8} = \frac{28.8 \sin N}{28.8}$

$0.755 = \sin N$

$\boxed{N = 49.0°}$

$180 - 65 - 49 = L$

$\boxed{L = 66°}$

4. $17^2 = 11^2 + 14^2 - 2(11)(14)\cos L$

$289 = 317 - 308\cos L$

$-28 = -308\cos L$

$\frac{-28}{-308} = \cos L$

$\cos^{-1}\left(\frac{-28}{-308}\right) = L$

$\boxed{L = 84.8°}$

$\frac{\sin 84.8}{17} = \frac{\sin M}{14}$

$\frac{14 \cdot \sin 84.8}{17} = \frac{17 \cdot \sin M}{17}$

$0.820 = \sin M$

$\boxed{M = 55.1°}$

$180 - 84.8 - 55.1 =$

$\boxed{N = 40.1}$

5. $l^2 = 31^2 + 64^2 - 2(31)(64)\cos 37°$

$l^2 = 961 + 4096 - 3968 \cos 37°$

$l^2 = 5057 - 3968 \cos 37°$

$l^2 = 1888.014$

$\boxed{l = 43.5}$

$\frac{\sin 37}{43.5} = \frac{\sin N}{64}$

$\frac{64 \cdot \sin 37}{43.5} = \frac{43.5 \sin N}{43.5}$

$0.885 = \sin N$

$\boxed{N = 62.3°}$

$180 - 37 - 62.3 = M$

$\boxed{M = 80.7°}$

6. $38^2 = 40^2 + 70^2 - 2(40)(70)\cos L$

$1444 = 1600 + 4900 - 5600 \cos L$

$-5056 = -5600 \cos L$

$\frac{-5056}{-5600} = \cos L$

$\cos^{-1}\left(\frac{-5056}{-5600}\right) = L$

$\boxed{L = 25.5°}$

$\frac{\sin 25.5}{38} = \frac{\sin N}{70}$

$\frac{70 \cdot \sin 25.5}{38} = \frac{38 \cdot \sin N}{38}$

$0.793 = \sin N$

$\boxed{N = 52.5°}$

$180 - 25.5 - 52.5 = M$

$\boxed{M = 102°}$

7. $15^2 = 7^2 + 12^2 - 2(7)(12)\cos M$

$225 = 49 + 144 - 168 \cos M$

$32 = -168 \cos M$

$\frac{32}{-168} = \cos M$

$\cos^{-1}\left(\frac{32}{-168}\right) = M$

$\boxed{M = 101°}$

$\frac{\sin 101}{15} = \frac{\sin N}{12}$

$\frac{12 \cdot \sin 101}{15} = \frac{15 \cdot \sin N}{15}$

$0.785 = \sin N$

$\boxed{N = 51.7°}$

$180 - 101 - 51.7 = L$

$\boxed{L = 27.3°}$

8. $m^2 = 23^2 + 29^2 - 2(23)(29)\cos 18°$

$m^2 = 529 + 841 - 1334 \cos 18$

$m^2 = 1370 - 1268.709$

$m^2 = 101.291$

$\boxed{m = 10.1}$

$\frac{\sin 18°}{10.1} = \frac{\sin L}{23}$

$\frac{23 \cdot \sin 18°}{10.1} = \frac{10.1 \sin L}{10.1}$

$0.704 = \sin L$

$\boxed{L = 44.7°}$

$180 - 18 - 44.7 = N$

$\boxed{N = 117.3°}$

9. $14^2 = 12^2 + 13^2 - 2(12)(13)\cos L$

$196 = 144 + 169 - 312\cos L$

$-117 = -312\cos L$

$\frac{-117}{-312} = \cos L$

$\cos^{-1}\left(\frac{-117}{-312}\right) = L$

$\boxed{L = 68°}$

$\frac{\sin 68}{14} = \frac{\sin M}{13}$

$\frac{13 \cdot \sin 68}{14} = \frac{14 \cdot \sin M}{14}$

$0.861 = \sin M$

$\boxed{M = 59.4°}$

$180 - 68 - 59.4 = N$

$\boxed{N = 52.6°}$

10. $b^2 = 63^2 + 91^2 - 2(63)(91)\cos 153.4$

$b^2 = 3969 + 8281 - 11466\cos 153.4$

$b^2 = 12,250 - (-10252.37)$

$b^2 = 22,502.37$

$b \approx 150$ miles

Plane travels at 300 mph.

Trip takes 30 min.

11. Skip