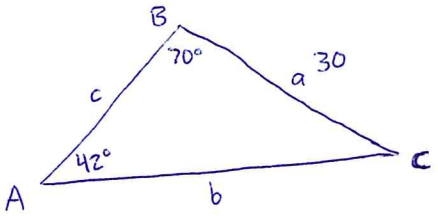
Algebra 2B----Law of Sines

Today I am going to teach you how to find side lengths and angle measures when you **don't** have a right triangle. There are two formulas, and today we will just look at one of them. It's called the "Law of Sines". Read on!!!

1. Draw a triangle below (but make sure it is NOT a right triangle). Make it big enough to label it and be able to refer back to it.



2. Label the Corners A, B and C (make sure to use capital letters).

3. Now put one finger on angle A. Good, now put another finger on the side that is opposite angle A (the side that doesn't touch A at all!!!). Label this side "a" (use a lower case letter this time.)

4. Do the same thing to label side "b" and side "c".

*** Notice that the capital letters stand for angles and the lower case letters stand for side lengths. ***

5. What do capital letters stand for? Angles

6. What do lower case letters stand for? Side Lengths

7. Here is the formula for the Law of Sines: $\boxed{\frac{\sin X}{x} = \frac{\sin Y}{y}}$

8. Hmmm, I put a capital X next to the first sine function…what does that mean capital X is? an angle

9. I put a lower case x on under the SinX,. What does the lower case x represent? a side length

10. So in general, the angle measures always go on the top of the fraction.

11. In general, the side lengths always go on the bottom of the fraction.

12. In your triangle way above, label angle A as 42 degrees and label angle B as 70 degrees. Then label side "a" as being 30 long. Use the formula to find side "b". Do this below!!

$$\frac{\sin 42}{30} = \frac{\sin 70}{b} \qquad \frac{b \cdot \sin 42}{\sin 42} = \frac{30 \cdot \sin 70}{\sin 42}$$

$b = 42.1$

13. Did you get 42.1? If so, move on. If not, get help right now!!!

*** Now try these few problems before moving on. Check your answers when you're done. ***

Find each measure using the given measures of ΔABC. Round angle measures to the nearest degree and side measures to the nearest tenth of a degree.

1. If c = 13, m∠A = 78, and m∠C = 38, find a.

$\frac{\sin 38}{13} = \frac{\sin 78}{a}$

$a \cdot \sin 38 = 13 \cdot \sin 78$

$a = 20.7$

2. If b = 22, c = 34, and m∠C = 62, find m∠B.

$\frac{\sin 62}{34} = \frac{\sin B}{22}$

$\frac{34 \cdot \sin B}{34} = \frac{22 \cdot \sin 62}{34}$

$\sin B = 0.5713$

$B = 34.8°$

3. If a = 19, c = 12, and m∠A = 88, find m∠C.

$\frac{\sin 88}{19} = \frac{\sin C}{12}$

$12 \cdot \sin 88 = 19 \cdot \sin C$

$\frac{12 \cdot \sin 88}{19} = \sin C$

$\sin C \approx 0.631194$

$\sin^{-1} \sin C = \sin^{-1}(0.631$

$C = 39.1°$

4. If a = 27, m∠A = 70, and m∠B = 15, find b.

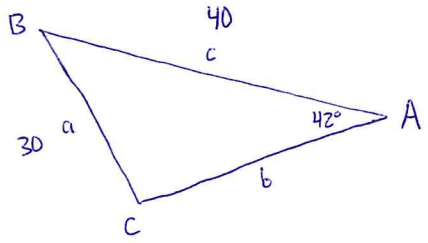
$\frac{\sin 70}{27} = \frac{\sin 15}{b}$

$\frac{b \cdot \sin 70}{\sin 70} = \frac{27 \cdot \sin 15}{\sin 70}$

$b = 7.4$

---

14. Make a new triangle below. Label the angles A,B, and C. Label the sides "a", "b", and "c". In your triangle below, label angle A as 42 degrees. Then label side "a" as being 30 long and label side "c" as being 40 long. Use the formula to find angle C. Go!!!



$\frac{\sin 42}{30} = \frac{\sin C}{40}$

$\frac{30 \cdot \sin C}{30} = \frac{40 \cdot \sin 42}{30}$

$\sin C = 0.89217$

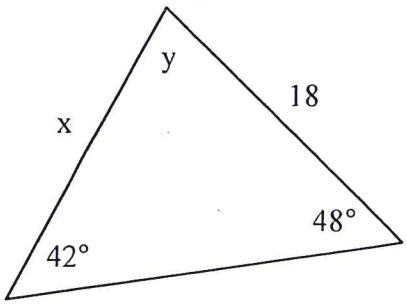
$C = 63.1°$

15. Did you get 63.1? If so, move on. If not, take your right arm and throw it up in the air. I will come help you. Do it now!!

*** Now finish the rest of the problems on the next few pages. ***

Use the given triangle and the information provided to solve for x and y.

1.



$y = 180 - 48 - 42$

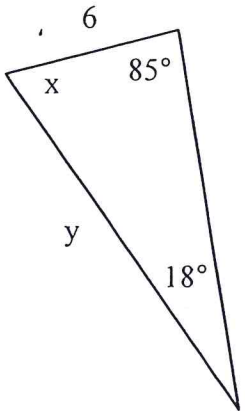
$y = 90°$

$\frac{\sin 42}{18} = \frac{\sin 48}{x}$

$\frac{x \cdot \sin 42}{\sin 42} = \frac{18 \cdot \sin 48}{\sin 42}$

$x = 20.0$

2.



$x = 180 - 85 - 18$

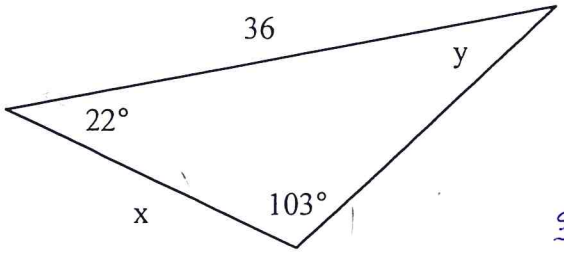
$x = 77°$

$\frac{\sin 18}{6} = \frac{\sin 85}{y}$

$\frac{y \cdot \sin 18}{\sin 18} = \frac{6 \cdot \sin 85}{\sin 18}$

$y = 19.3$

3.



$y = 180 - 103 - 22$

$y = 55°$

$\frac{\sin 103}{36} = \frac{\sin 55}{x}$

$\frac{x \cdot \sin 103}{\sin 103} = \frac{36 \cdot \sin 55}{\sin 103}$

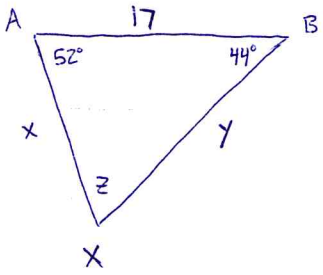
$x = 30.3$

Draw a triangle to go with each problem and label it with the given information. Then solve for what you are asked. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. Two ice-cream stores A and B are 17 miles apart. Bill is located at point X at the same time. The three points form ∠XAB, which measures 52°, and ∠XBA, which measures 44°. How far is Bill from each ice-cream store?

$z = 180 - 52 - 44$

$z = 84°$



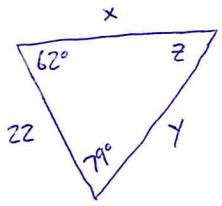
$\frac{\sin 84}{17} = \frac{\sin 44}{x}$ $x = 11.9$

$\frac{\sin 84}{17} = \frac{\sin 52}{y}$ $y = 13.5$

2. One side of Greg's triangular back yard is 22 feet. The angles on each side of this side measure 62° and 79°. Find the total length of fence needed to enclose the garden.

$z = 180 - 79 - 62$

$z = 39°$



$\frac{\sin 39}{22} = \frac{\sin 79}{x}$ $x = 34.3$

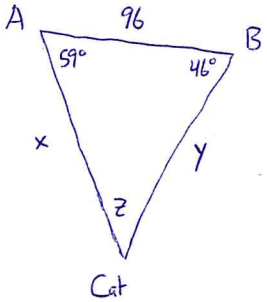
$\frac{\sin 39}{22} = \frac{\sin 62}{y}$ $y = 30.9$

Total: 34.3 + 30.9 + 22 = 87.2

3. A frightened pet owner wants to determine the distances from points A and B to her cat that is stuck in a tree. ∠BAC measures 59°. ∠ABC measures 46°. If points A and B are 96 feet apart, find the distance from C to each point.

$z = 180 - 59 - 46$

$z = 75°$



$\frac{\sin 75}{96} = \frac{\sin 46}{x}$ $x = 71.5$

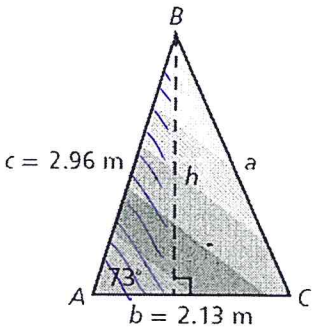
$\frac{\sin 75}{96} = \frac{\sin 59}{y}$ $y = 85.2$

Area of a Triangle

All of you should know that the formula for area of a triangle is $A = \frac{1}{2}bh$. However, what if you did not know what the height of the triangle was? Can you still solve this type of problem if you're given other pieces of information about the triangle?

Example #1

Look at the triangle to the right. We want to find the area of that triangle, but we don't know the height. What we're going to have to do is use trig. functions to represent h.



Use the information you've been given in the triangle to represent h using trig. functions. **TRY IT!!**

$\sin A = \frac{h}{c}$ $\frac{(\text{opposite})}{(\text{hypotenuse})}$

You should (*hopefully*) get to this: $\sin A = \frac{h}{c}$

Solving this for "h" would then get us $h = c \sin A$

Now we're going to plug this "h" into our original area formula of a triangle.

$$A = \frac{1}{2}bh \;\rightarrow\rightarrow\rightarrow\; A = \frac{1}{2}b(c \sin A)$$

Using the formula, find the area of the triangle above.

$A = \frac{1}{2}(2.13)(2.96 \cdot \sin 73^\circ)$

Answer = 3.01 Check with a neighbor or me!!!!

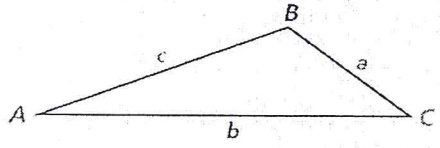
Area of a Triangle

For $\triangle ABC$,

Area $= \frac{1}{2}bc \sin A$

Area $= \frac{1}{2}ac \sin B$

Area $= \frac{1}{2}ab \sin C$



Name _____ Date _____ Class _____

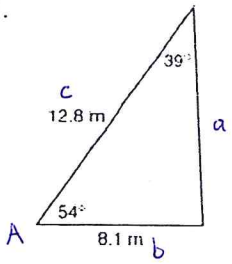
LESSON 13-5

Practice B

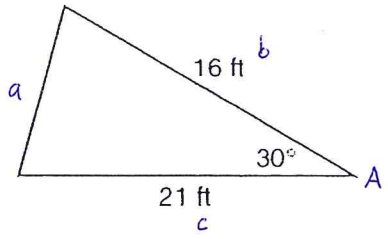
The Law of Sines

Find the area of each triangle. Round to the nearest tenth.

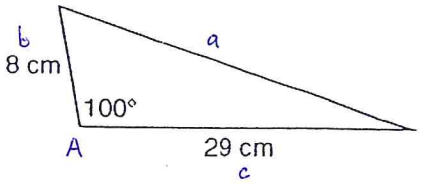
1.



2.

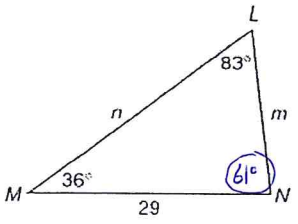


3.

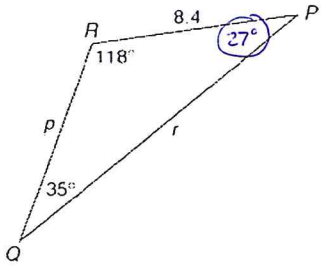


Solve each triangle. Round to the nearest tenth.

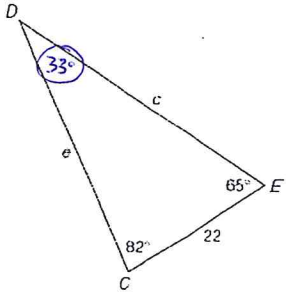
4.



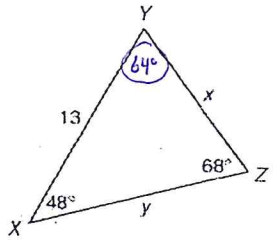
5.



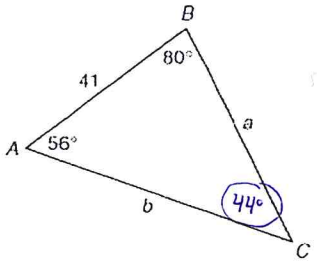
6.



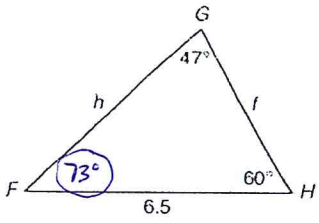
7.



8.



9.



Solve.

13. Margaret has two lengths of fence, 20 meters and 24 meters, for two sides of a triangular chicken pen. The third side will be on the north side of a barn. One fence length forms a 75° angle with the barn. How many different pens can she build if one fence is attached at the corner of the barn? What are all the possible lengths for the barn side of the pen?

Solve.

13. Ann is creating a triangular frame. Two angles and the included side of the frame measure 64°, 58°, and 38 centimeters, respectively. What are the lengths of the other two sides of the frame to the nearest tenth of a centimeter?

Practice B Worksheet

1. $A = \frac{1}{2} b (c \sin A)$

$= \frac{1}{2}(8.1)(12.8 \cdot \sin 54)$

$A = 41.9\ m^2$

2. $A = \frac{1}{2} b (c \sin A)$

$= \frac{1}{2}(16)(21 \cdot \sin 30)$

$A = 84\ ft^2$

3. $A = \frac{1}{2} b (c \cdot \sin A)$

$= \frac{1}{2}(8)(29 \cdot \sin 100)$

$A = 114.2\ cm^2$

4. $\frac{\sin 83}{29} = \frac{\sin 61}{n}$ $n = 25.6$

$\frac{\sin 83}{29} = \frac{\sin 36}{m}$ $m = 17.2$

5. $\frac{\sin 35}{8.4} = \frac{\sin 27}{p}$ $p = 6.6$

$\frac{\sin 35}{8.4} = \frac{\sin 118}{r}$ $r = 12.9$

6. $\frac{\sin 33}{22} = \frac{\sin 82}{c}$ $c = 40.0$

$\frac{\sin 33}{22} = \frac{\sin 65}{e}$ $e = 36.6$

7. $\frac{\sin 68}{13} = \frac{\sin 48}{x}$ $x = 10.4$

$\frac{\sin 68}{13} = \frac{\sin 64}{y}$ $y = 12.6$

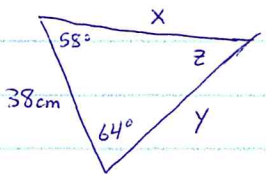
8. $\frac{\sin 44}{41} = \frac{\sin 56}{a}$ $a = 48.9$

$\frac{\sin 44}{41} = \frac{\sin 80}{b}$ $b = 58.1$

9. $\frac{\sin 47}{6.5} = \frac{\sin 73}{f}$ $f = 8.5$

$\frac{\sin 47}{6.5} = \frac{\sin 60}{h}$ $h = 7.7$

13.



$z = 180 - 64 - 58$

$z = 58°$

$\frac{\sin 58}{38} = \frac{\sin 64}{y}$ $y = 40.3\ cm$

$\frac{\sin 58}{38} = \frac{\sin 58}{x}$ $x = 38\ cm$