

LESSON 7-5 **Reteach**
Exponential and Logarithmic Equations and Inequalities

An **exponential equation** contains an expression that has a variable as an exponent.

$5^x = 25$ is an exponential equation.
 $x = 2$, since $5^2 = 25$.

If $x = y$, then
 $\log x = \log y$
 $(x > 0 \text{ and } y > 0)$.

Remember: You can take the logarithm of both sides of an exponential equation. Then use other properties of logarithms to solve.

Solve $6^{x+2} = 500$.

Step 1 Since the variable is in the exponent, take the log of both sides.

$6^{x+2} = 500$
 $\log 6^{x+2} = \log 500$

Step 2 Use the Power Property of Logarithms: $\log a^p = p \log a$.

$\log 6^{x+2} = \log 500$
 $(x + 2) \log 6 = \log 500$ "Bring down" the exponent to multiply.

Step 3 Isolate the variable. Divide both sides by log 6.

$(x + 2) \log 6 = \log 500$
 $x + 2 = \frac{\log 500}{\log 6}$

Step 4 Solve for x. Subtract 2 from both sides.

$x = \frac{\log 500}{\log 6} - 2$

Step 5 Use a calculator to approximate x.

$x \approx 1.468$

Step 6 Use a calculator to check.

$6^{1.468+2} \approx 499.607$

Solve and check.

1. $4^{-x} = 32$
 $\log 4^{-x} = \log 32$
 $-x \log 4 = \log 32$
 $-x = \frac{\log 32}{\log 4}$

 $x = -2.5$

2. $3^{4x} = 90$
 $\log 3^{4x} = \log 90$
 $4x \log 3 = \log 90$
 $4x = \frac{\log 90}{\log 3}$

 $x = 1.02$

3. $5^{x-3} = 600$
 $\log 5^{x-3} = \log 600$
 $(x-3) \log 5 = \log 600$
 $x-3 = \frac{\log 600}{\log 5}$

 $x = 6.97$

LESSON
7-5

Reteach

Exponential and Logarithmic Equations and Inequalities (continued)

A **logarithmic equation** contains a logarithmic expression that has a variable.

$$\log_5 x = 2 \text{ is a logarithmic equation.}$$

$$x = 25, \text{ since } 5^2 = 25.$$

Combine and use properties of logarithms to solve logarithmic equations.

Solve: $\log 80x - \log 4 = 1$

Step 1 Use the Quotient Property of Logarithms.

$$\log 80x - \log 4 = 1$$

$$\log \frac{80x}{4} = 1$$

$\log x - \log y = \log \frac{x}{y}$

Step 2 Simplify.

$$\log \frac{80x}{4} = 1$$

$$\log 20x = 1$$

Step 3 Use the definition of the logarithm:

if $b^x = a$, then $\log_b a = x$.

$$\log_{10} 20x = 1$$

$$10^1 = 20x$$

Remember: Use 10 as the base when the base is not given.

Step 4 Solve for x . Divide both sides by 20.

$$10 = 20x$$

$$\frac{1}{2} = x$$

Solve and check.

4. $\log_3 x^4 = 8$

$$4 \log_3 x = 8$$

$$\log_3 x = \frac{8}{4}$$

$$\underline{3^2 = x}$$

$$\underline{x = 9}$$

5. $\log 4 + \log (x + 2) = 2$

$$\log 4(x + 2) = 2$$

$$\log_{10} (4x + 8) = 2$$

$$4x + 8 = 10^2$$

$$\underline{4x + 8 = 100}$$

$$\underline{x = 23}$$

6. $\log 75x - \log 3 = 1$

$$\log \frac{75x}{3} = 1$$

$$\log_{10} 25x = 1$$

$$10^1 = 25x$$

$$\underline{x = 10/25}$$

$$\underline{x = 2/5}$$

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Mr. Ward Answer Key

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$$2. 4^{2x} = 32^{1/2}$$

$$\log 4^{2x} = \log 32^{1/2}$$

$$\frac{2x \log 4}{\log 4} = \frac{\frac{1}{2} \log 32}{\log 4}$$

$$2x \cdot 2x = \frac{1}{2} \left(\frac{\log 32}{\log 4} \right) \times 2$$

$$4x = \frac{\log 32}{\log 4}$$

$$4x \approx 2.5$$

$$x = 0.625$$

$$3. 9^x = 3^{x-2}$$

$$\log 9^x = \log 3^{x-2}$$

$$x \frac{\log 9}{\log 3} = \frac{(x-2) \log 3}{\log 3}$$

$$x \left(\frac{\log 9}{\log 3} \right) = x - 2$$

$$x(2) = x - 2$$

$$\frac{2x}{-x} = \frac{x-2}{-x}$$

$$x = -2$$

$$4. 2^x = 4^{x+1}$$

$$\log 2^x = \log 4^{x+1}$$

$$x \frac{\log 2}{\log 4} = \frac{(x+1) \log 4}{\log 4}$$

$$x \left(\frac{\log 2}{\log 4} \right) = x + 1$$

$$x \left(\frac{1}{2} \right) = x + 1$$

$$\frac{1}{2}x = x + 1$$

$$\frac{-1}{2}x = 1$$

$$x = -2$$

$$x = -2$$

$$5. 4^x = 10$$

$$\log 4^x = \log 10$$

$$x \frac{\log 4}{\log 4} = \frac{\log 10}{\log 4}$$

$$x = 1.66$$

$$6. \left(\frac{1}{4} \right)^{2x} = \left(\frac{1}{2} \right)^x$$

$$\log \left(\frac{1}{4} \right)^{2x} = \log \left(\frac{1}{2} \right)^x$$

$$2x \frac{\log \left(\frac{1}{4} \right)}{\log \left(\frac{1}{4} \right)} = x \frac{\log \left(\frac{1}{2} \right)}{\log \left(\frac{1}{4} \right)}$$

$$2x = x \left(\frac{1}{2} \right)$$

$$2x = \frac{1}{2}x$$

Can only work if

$$x = 0$$

$$7. 2.4^{3x+1} = 9$$

$$\log 2.4^{3x+1} = \log 9$$

$$(3x+1) \frac{\log 2.4}{\log 2.4} = \frac{\log 9}{\log 2.4}$$

$$3x+1 \approx 2.5098$$

$$3x = 1.5098$$

$$x = 0.503$$

$$8. P = 3400(1+0.03)^t$$

- Make a table and start plugging in values for t.

- Keep trying numbers until you get a P value of 10,000+

Answer = 37 years

$$9. \log_2(7x+1) = \log_2(2-x)$$

$$\frac{7x+1}{+x} = \frac{2-x}{+x}$$

$$8x+1 = 2$$

$$\frac{8x}{8} = \frac{1}{8}$$

$$x = 1/8$$

$$10. \log_6(2x+3) = 3$$

$$6^3 = 2x+3$$

$$216 = 2x+3$$

$$213 = 2x$$

$$106.5 = x$$

$$11. \log 72 - \log \left(\frac{2x}{3}\right) = 0$$

$$+ \log \left(\frac{2x}{3}\right) + \log \left(\frac{2x}{3}\right)$$

$$\log 72 = \log \frac{2x}{3}$$

$$3 \times 72 = \left(\frac{2x}{3}\right) \times 3$$

$$216 = 2x$$

$$108 = x$$

$$12. \log_3 x^9 = 12$$

$$\frac{9 \log_3 x}{9} = \frac{12}{9}$$

$$\log_3 x = \frac{4}{3}$$

$$3^{(4/3)} = x$$

$$4.33 \approx x$$

$$13. \log_7 (3-4x) = \log_7 \left(\frac{x}{3}\right)$$

$$\times 3 (3-4x) = \left(\frac{x}{3}\right) \times 3$$

$$9 - 12x = x$$

$$\frac{9}{13} = \frac{13x}{13}$$

$$\frac{9}{13} = x$$

$$14. \log 50 + \log \left(\frac{x}{2}\right) = 2$$

Product Property

$$\log (50 \cdot \frac{x}{2}) = 2$$

$$10^2 = \frac{50x}{2}$$

$$\times 2 (100) = \left(\frac{50x}{2}\right) \times 2$$

$$200 = 50x$$

$$4 = x$$

$$15. \log x + \log (x+48) = 2$$

Product Property

$$\log x(x+48) = 2$$

$$10^2 = x(x+48)$$

$$100 = x^2 + 48x$$

Quadratic so set = to 0

$$0 = x^2 + 48x - 100$$

$$0 = (x+50)(x-2)$$

$$x+50 = 0$$

$$-50 \quad -50$$

$$x-2 = 0$$

$$+2 \quad +2$$

$$x = -50$$

$$x = 2$$