<u>The Natural Base "e"</u>

One very important number that arises when we are dealing with exponential and logarithmic functions is the "natural" exponential. This number is denoted as the letter "e". (It can be found on your calculator! Important!)

We've already learned that logarithmic functions are the inverses of exponential functions. It should be no surprise then that the inverse of this "natural" exponential function is the "natural" logarithmic function: *ln x*.

While regular logarithms were in base 10, <u>the base for the natural logarithm is the number "e"</u>.

<u>All the properties of logarithms that you've already learned still apply!</u> In addition to those properties, here are some properties you will also need to know specifically for the natural base e.

When looking at these properties keep in mind that the base for the natural logarithm is ______________.

- *ln* 1 = 0 *(Obviously $e^0 = 1$ because anything to the 0 power equals 1)*
- *ln* e = 1 *(Pretty obvious again, $e^1 = e$)*

<u>Example #1</u> *ln* 2 + *ln* 3x = 4.7

Step 1: Just like with logarithms, you will need to use a logarithmic property to simplify those two natural logarithms into <u>one</u> natural logarithm.

Step 2: Now that the logarithm part is by itself, convert it into exponential form.

(Answer: $e^{4.7} = 6x$)

Step 3: Use your calculator to multiply out the exponent part. Then solve for x.

<u>Example #2</u> $e^{x-4} = 24$

The thing to notice here is that this problem started with exponential form. (*Unlike Example #1 that started with logarithmic form.*) Therefore in order to solve this we're going to have to convert to logarithmic form. I told you switching between these two forms was going to be important!

Step 1: We're going to convert this "natural" exponential into a "natural" logarithm. Do this below:

(Answer: *ln* 24 = x – 4)

Step 2: Now we're going to use our calculator. On your calculator there is a button for *ln*. Plug in *ln* 24 into your calculator and get a value.

(Answer: 3.178 = x – 4)

Step 3: Solve for x now.

Try these practice problems now. Good luck! ☺

1. $\ln x - \ln 5 = 3$

2. $\ln 5 + \ln x = 1$

3. $e^{x} = 2$

4. $e^{-3y} = 83$

5. $\ln 10 + \ln x^{2} = 10$

6. $2 \cdot \ln x - 2 = 0$

7. $e^{n+7} = 26$

8. $9 \cdot e^{1.4x - 10} - 10 = 17$

9. $\ln (1 - 8n) - 10 = -7$

10. $-8 \cdot \ln -9x = -8$

11. $\ln 5 + \ln (4 - 5x) = 3$

12.* $\ln (5 - 2x^{2}) + \ln 9 = \ln 43$

Name ______________________ Date ____________ Class ____________

LESSON 7-6

Reteach

The Natural Base, e (continued)

The natural base, e, appears in the formula for interest compounded continuously.

$A = Pe^{rt}$

A = total amount

P = principal, or initial amount

r = annual interest rate

t = time in years

What is the total amount for an investment of $2000 invested at 3% and compounded continuously for 5 years?

Step 1 Identify the values that correspond to the variables in the formula.

P = initial investment = $2000

$r = 3\% = 0.03$

$t = 5$

Step 2 Substitute the known values into the formula.

$A = Pe^{rt}$

$A = 2000\, e^{0.03\,(5)}$

Step 3 Use a calculator to solve for A, the total amount.

$A = 2000\, e^{0.03\,(5)}$

$A \approx 2323.67$

Use the e^x key on a calculator: $2000e^{(.03*5)} = 2323.668485$

The total amount is $2323.67.

Use the formula $A = Pe^{rt}$ to solve.

7. What is the total amount for an investment of $500 invested at 4.5% and compounded continuously for 10 years?

   P = ____________ r = ____________ t = ____________

   ____

8. Randy deposited $1000 into an account that paid 2.8% with continuous compounding. What was her balance after 6 years? ____________

9. a. Martin borrows $5500. The rate is set at 6% with continuous compounding. How much does he owe at the end of 2 years? ____________

   b. Martin found a bank with a better interest rate of 5.5%. How much less does he owe at the end of 2 years? ____________

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LESSON 7-6

Practice A

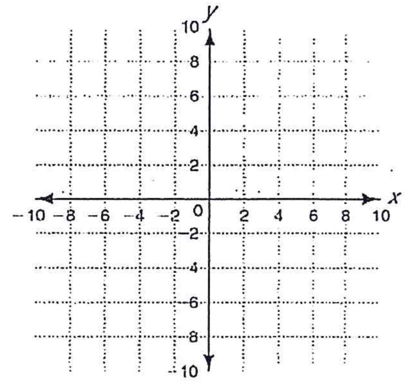
The Natural Base, e

Graph each exponential function.

1. $f(x) = e^{-x}$

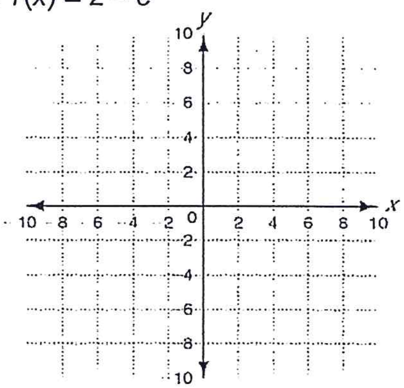
   a. Complete the table.

x	−2	−1	0	1	2	3
$f(x)$	7.4					

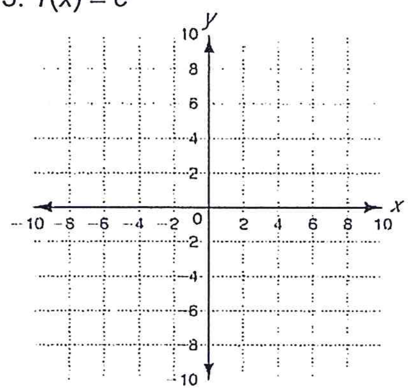


   b. Graph the ordered pairs and draw a curve through the points.

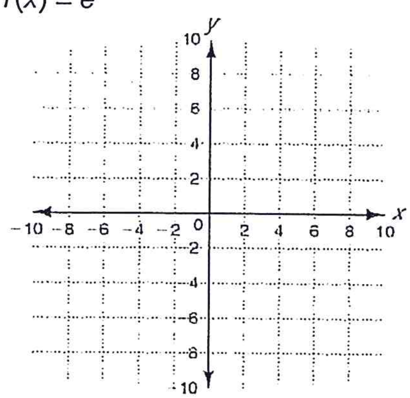
2. $f(x) = 2 - e^{x}$



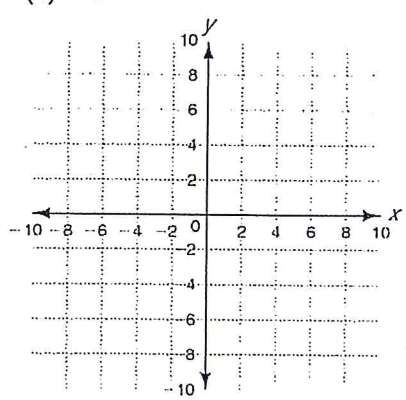
3. $f(x) = e^{2-x}$



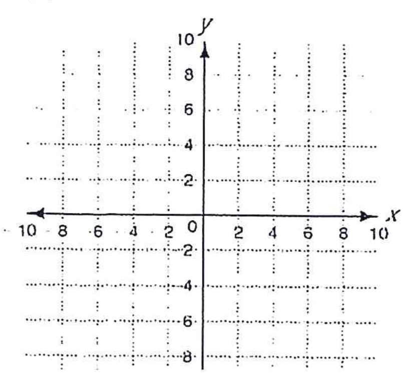
1. $f(x) = e^{2x}$



2. $f(x) = e^{0.5x}$



3. $f(x) = e^{1+x}$



4. $f(x) = e^{2-x}$