Mr. Ward Answer Key

Permutations and Combinations

Fundamental Counting Principle

The Fundamental Counting Principle is used when you have to make several choices out of a group of objects. For example, let's say you are making an ice cream sundae. There are 5 different kinds of ice cream, 4 different toppings, and 2 different types of sprinkles. How many different possible sundaes' are there?

To do this we just set up a simple multiplication problem: 5 * 4 * 2 = 40

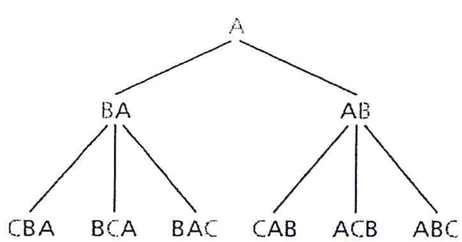
Practice Problem #1

A restaurant has a lunch special where you can pick from 6 entrées, 4 side dishes, and 9 drinks. How many different meal choices are there?

6 * 4 * 9 = 216

Permutations

We use permutations when we want to find out the total number of ways to order objects. For example, if we wanted to order the letters A, B, and C. How many ways could we do it? Keep in mind that AB is NOT the same as BA.



By looking at this chart we find that there are 6 different ways to order the letters A, B, and C.

When we are doing permutations, **order does matter!** (AB is different from BA.)

Practice Problem #2

There are 7 sprinters in the 100-yard dash. In how many ways can the top 3 finish?

Now making a flow chart for this would become pretty difficult. Let's look at a formula we can use to make our lives easier.

$${}_nP_r = \frac{n!}{(n-r)!}$$

This formula represents the number of permutations of n items taken r at a time.

So for our example it would look like this:

$${}_7P_3 = \frac{7!}{(7-3)!} \rightarrow \frac{7 \cdot 6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} \rightarrow 7 \cdot 6 \cdot 5 = \underline{\quad 210 \quad}$$

Combinations

Lastly, we use combinations when we want to group items together. Unlike with permutations, **order does not matter!**

For example the group of ABC is the same as BAC, CAB, ACB, etc.

This means that when we deal with combinations there will be fewer ways to select items. Here is the formula we will use for combinations. It's very similar to the permutation formula but there are some slight differences.

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

Practice Problem #3

Katie is going to adopt kittens from a litter of 11. How many ways can she choose a group of 3 kittens?

Step 1: We first have to decide whether this is a permutation or a combination problem. The question you have to ask yourself is, does order matter?

In this example order does not matter. The group of kittens, Smokey, Tigger, and Kitty is the same group as Tigger, Kitty, and Smokey.

Step 2: Therefore we will use the combination formula.

$${}_{11}C_3 = \frac{11!}{3!(11-3)!} \rightarrow \frac{11 \cdot 10 \cdot 9 \cdot \cancel{8} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{3 \cdot 2 \cdot 1 \cdot \cancel{8} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} \rightarrow \frac{11 \cdot 10 \cdot 9}{3 \cdot 2 \cdot 1} = \underline{\quad 165 \quad}$$

Come show me your answer so I know you're on the right track!

You should now be able to do the following things:

- I can apply and use the Fundamental Counting Principle.
- I can apply and use Permutations.
- I can apply and use Combinations.
- I can solve problems using Permutations & Combinations.

Assignment: Practice A Worksheet + Practice B Worksheet

Name Mr. Ward Date ________ Class ________

LESSON 11-1

Practice A

Permutations and Combinations

Use the Fundamental Counting Principle.

1. For her aquarium, Susan can choose from 4 types of fish and 3 types of plants. If she chooses one type of fish and one type of plant, how many different aquariums can Susan set up?

   $4 \cdot 3 = \boxed{12}$

2. Lottery numbers in a particular state consist of 6 digits. Each lottery ball that is drawn has six sides, numbered 1–6. How many different lottery numbers are possible?

   $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = \boxed{46,656}$

Evaluate.

3. $5!$

   $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{120}$

4. $\frac{6!}{2!}$ $\quad \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot \cancel{1}}$

   $= \boxed{360}$

5. $\frac{2!3!}{4!}$ $\quad \frac{2 \cdot 1 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$

   $= \boxed{\frac{1}{2}}$

Solve.

6. For an art exhibit, Craig has to choose 3 ceramic mugs out of the 7 that he made over the summer. In how many ways can he arrange these 3 mugs in a row? (order does matter)

   ${}_7P_3 = \frac{7!}{(7-3)!} = \frac{7 \cdot 6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$

   $= \boxed{210 \text{ ways}}$

7. The members of a track team want to choose a team captain and someone to organize their equipment. In how many ways can 2 people be chosen from a team of 10 girls? (order does matter)

   ${}_{10}P_2 = \frac{10!}{(10-2)!} = \frac{10 \cdot 9 \cdot \cancel{8} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{8} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$

   $= \boxed{90 \text{ ways}}$

Evaluate.

8. ${}_4P_3$ $\quad \frac{4!}{(4-3)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{1}$

   $= \boxed{24}$

9. ${}_3C_2$ $\quad \frac{3!}{2!(3-2)!} \quad \frac{3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 1}$

   $= \boxed{3}$

10. ${}_5P_2$ $\quad \frac{5!}{(5-2)!} \quad \frac{5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$

    $= \boxed{20}$

11. ${}_5C_2$ $\quad \frac{5!}{2!(5-2)!} \quad \frac{5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{2 \cdot 1 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$

    $= \boxed{10}$

12. ${}_8P_4$ $\quad \frac{8!}{(8-4)!} \quad \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$

    $= \boxed{1680}$

13. ${}_6C_1$ $\quad \frac{6!}{1!(6-1)!} \quad \frac{6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{1 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$

    $= \boxed{6}$

Solve.

14. While on vacation, Sandra wants to buy 2 wallets. There are 7 different patterns she can choose from. In how many ways can Sandra choose 2 different wallets? (order does not matter)

    ${}_7C_2 = \frac{7!}{2!(7-2)!} = \frac{7 \cdot 6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{2 \cdot 1 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$

    $= \boxed{21 \text{ ways}}$

15. Vince chooses 3 side dishes from a total of 10 side dishes offered on the menu. In how many different ways can he choose his side dishes? (order does not matter)

    ${}_{10}C_3 = \frac{10!}{3!(10-3)!} = \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{3 \cdot 2 \cdot 1 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$

    $= \boxed{120 \text{ ways}}$

Name Mr. Ward Date ______________ Class ______________

LESSON 11-1

Practice B

Permutations and Combinations

Use the Fundamental Counting Principle.

1. The soccer team is silk-screening T-shirts. They have 4 different colors of T-shirts and 2 different colors of ink. How many different T-shirts can be made using one ink color on a T-shirt? $4 \cdot 2 = \boxed{8 \text{ T-shirts}}$

2. A travel agent is offering a vacation package. Participants choose the type of tour, a meal plan, and a hotel class from the table below.

Tour	Meal	Hotel
Walking	Restaurant	4-Star
Boat	Picnic	3-Star
Bicycle		2-Star
		1-Star

How many different vacation packages are offered? $3 \cdot 2 \cdot 4 = \boxed{24 \text{ packages}}$

Evaluate.

3. $\dfrac{3!6!}{3!}$ $\dfrac{\cancel{3\cdot 2\cdot 1}\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{\cancel{3\cdot 2\cdot 1}}$ $= \boxed{720}$

4. $\dfrac{10!}{7!}$ $\dfrac{10\cdot 9\cdot 8\cdot \cancel{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}}{\cancel{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}}$ $= \boxed{720}$

5. $\dfrac{9!-6!}{(9-6)!}$ $\dfrac{362{,}880 - 720}{3\cdot 2\cdot 1}$ $= \boxed{60{,}360}$

Solve.

6. In how many ways can the debate team choose a president and a secretary if there are 10 people on the team? (order does matter)
$_{10}P_2 = \dfrac{10!}{(10-2)!} = \dfrac{10\cdot 9\cdot \cancel{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}}{\cancel{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}}$ $= \boxed{90 \text{ ways}}$

7. A teacher is passing out first-, second-, and third-place prizes for the best student actor in a production of *Hamlet*. If there are 14 students in the class, in how many different ways can the awards be presented? (order does matter)
$_{14}P_3 = \dfrac{14!}{(14-3)!} = \dfrac{14\cdot 13\cdot 12\cdot \cancel{11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}}{\cancel{11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}}$ $= \boxed{2184 \text{ ways}}$

Evaluate.

8. $_5P_4$ $\dfrac{5!}{(5-4)!}$ $\dfrac{5\cdot 4\cdot 3\cdot 2\cdot 1}{1}$ $= \boxed{120}$

9. $_3C_2 = \dfrac{3!}{2!(3-2)!} = \dfrac{3\cdot \cancel{2\cdot 1}}{\cancel{2\cdot 1}\cdot 1}$ $= \boxed{3}$

10. $_8P_3 = \dfrac{8!}{(8-3)!} = \dfrac{8\cdot 7\cdot 6\cdot \cancel{5\cdot 4\cdot 3\cdot 2\cdot 1}}{\cancel{5\cdot 4\cdot 3\cdot 2\cdot 1}}$ $= \boxed{336}$

Solve.

11. Mrs. Marshall has 11 boys and 14 girls in her kindergarten class this year.

a. In how many ways can she select 2 girls to pass out a snack? $_{14}C_2 = \dfrac{14!}{2!(14-2)!} = \dfrac{14\cdot 13}{2\cdot 1}$ 91 ways

b. In how many ways can she select 5 boys to pass out new books? $_{11}C_5 = \dfrac{11!}{5!(11-5)!} = \dfrac{11\cdot 10\cdot 9\cdot 8\cdot 7}{5\cdot 4\cdot 3\cdot 2\cdot 1}$ 462 ways

c. In how many ways can she select 3 students to carry papers to the office? $_{25}C_3 = \dfrac{25!}{3!(25-3)!} = \dfrac{25\cdot 24\cdot 23}{3\cdot 2\cdot 1}$ 2300 ways