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11. $x^3 + 6x^2 - 5x - 30 = 0$

$$\frac{p}{q} = \frac{\pm 1 \ \pm 30 \ \pm 2 \ \pm 15 \ \pm 5 \ \pm 6 \ \pm 3 \ \pm 10}{\pm 1} = \pm 1 \ \pm 30 \ \pm 2 \ \pm 15 \ \pm 5 \ \pm 6 \ \pm 3 \ \pm 10$$

Graph it! $\longrightarrow$

$$\begin{array}{r|rrrr} -6 & 1 & 6 & -5 & -30 \\ & \downarrow & -6 & 0 & 30 \\ \hline & 1 & 0 & -5 & \boxed{0} \end{array}$$

$x^2 - 5 = 0$

$\sqrt{x^2} = \sqrt{5}$

$$\boxed{\begin{array}{c} x = \pm\sqrt{5} \\ x = -6 \end{array}}$$

12. $3x^3 - 18x^2 - 9x + 132 = 0$

$$\frac{p}{q} = \frac{\pm 1 \ \pm 132 \ \pm 2 \ \pm 66 \ \pm 4 \ \pm 33 \ \pm 6 \ \pm 22 \ \pm 3 \ \pm 44}{\pm 1 \ \ \pm 3}$$

$= \pm 1 \ \pm 132 \ \pm 2 \ \pm 66 \ \pm 4 \ \pm 33 \ \pm 6 \ \pm 22 \ \pm 3 \ \pm 44 \ \pm 1/3 \ \pm 2/3 \ \pm 4/3 \ \pm 11 \ \pm 22/3 \ \pm 44/3$

Graph it! $\longrightarrow$

$$\begin{array}{r|rrrr} 4 & 3 & -18 & -9 & 132 \\ & \downarrow & 12 & -24 & -132 \\ \hline & 3 & -6 & -33 & \boxed{0} \end{array}$$

$3x^2 - 6x - 33 = 0$

$$x = \frac{6 \pm \sqrt{36 - 4(3)(-33)}}{2(3)}$$

$$= \frac{6 \pm \sqrt{36 + 396}}{6}$$

$$= \frac{6 \pm \sqrt{432}}{6}$$

$$= \frac{6 \pm \sqrt{144 \cdot 3}}{6}$$

$$= \frac{6 \pm 12\sqrt{3}}{6}$$

$$\boxed{\begin{array}{c} x = 1 \pm 2\sqrt{3} \\ x = 4 \end{array}}$$

13. $2x^3 - 42x + 40 = 0$

$2(x^3 - 21x + 20)$

$\frac{p}{q} = \frac{\pm 1 \pm 20 \pm 4 \pm 5 \pm 2 \pm 10}{\pm 1} = \pm 1 \pm 20 \pm 4 \pm 5 \pm 2 \pm 10$

Graph it! $\longrightarrow$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -21 & 20 \\ & \downarrow & 1 & 1 & -20 \\ \hline & 1 & 1 & -20 & \boxed{0} \end{array}$$

$x^2 + x - 20$

$(x+5)(x-4)$

$\boxed{x = -5 \quad x = 4 \quad x = 1}$

14. $x^4 - 9x^2 + 20 = 0$

$\frac{p}{q} = \frac{\pm 1 \pm 20 \pm 4 \pm 5 \pm 2 \pm 10}{\pm 1} = \pm 1 \pm 20 \pm 4 \pm 5 \pm 2 \pm 10$

Graph it! $\longrightarrow$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -9 & 0 & 20 \\ & \downarrow & 2 & 4 & -10 & -20 \\ \hline & 1 & 2 & -5 & -10 & \boxed{0} \end{array}$$

$x^3 + 2x^2 - 5x - 10$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -5 & -10 \\ & \downarrow & -2 & 0 & 10 \\ \hline & 1 & 0 & -5 & \boxed{0} \end{array}$$

$x^2 - 5 = 0$

$\sqrt{x^2} = \sqrt{5}$

$\boxed{x = \pm\sqrt{5} \quad x = 2 \quad x = -2}$

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15. $x^3 + 3x^2 - 9x = 27$

$x^3 + 3x^2 - 9x - 27 = 0$

$x^2(x+3) - 9(x+3) = 0$

$(x^2 - 9)(x+3)$

$(x+3)(x-3)(x+3)$

$\boxed{x = -3 \quad x = 3 \quad x = -3}$

16. $4x^5 - 8x^3 + 4x = 0$

$4x(x^4 - 2x^2 + 1) = 0$

$4x(x^2-1)(x^2-1) = 0$

$4x(x-1)(x+1)(x-1)(x+1) = 0$

$\boxed{x=0 \quad x=1 \quad x=-1 \quad x=1 \quad x=-1}$

17. $10x^3 - 640x = 0$

$10x(x^2 - 64) = 0$

$10x(x-8)(x+8) = 0$

$\boxed{x=0 \quad x=8 \quad x=-8}$

18. $x^4 - 12x^2 = -36$

$x^4 - 12x^2 + 36 = 0$

$(x^2-6)(x^2-6) = 0$

$x^2 - 6 = 0$

$+6 \quad +6$

$\sqrt{x^2} = \sqrt{6}$ $\boxed{x = \pm\sqrt{6} \quad x = \pm\sqrt{6}}$

19. $2x^3 - 5x^2 - 4x + 10 = 0$

$x^2(2x-5) - 2(2x-5) = 0$

$(x^2-2)(2x-5) = 0$

$\boxed{x = \pm\sqrt{2}}$ $2x - 5 = 0$

$2x = 5$

$\boxed{x = 5/2}$

20. $4x^3 + 7x^2 - 5x = 6$

$4x^3 + 7x^2 - 5x - 6 = 0$

$\frac{p}{q} = \frac{\pm1 \ \pm2 \ \pm3 \ \pm6}{\pm1 \ \pm2 \ \pm4} = \pm1 \ \pm2 \ \pm3 \ \pm6 \ \pm1/2 \ \pm3/2 \ \pm1/4 \ \pm3/4$

Graph it!

-2	4	7	-5	-6
	↓	-8	2	6
	4	-1	-3	0

$4x^2 - x - 3 = 0$

$$x = \frac{1 \pm \sqrt{1 - 4(4)(-3)}}{2(4)}$$

$$= \frac{1 \pm \sqrt{1 + 48}}{8}$$

$$= \frac{1 \pm \sqrt{49}}{8}$$

$$= \frac{1 \pm 7}{8}$$

$\boxed{x = 1, -3/4 \quad x = -2}$

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24. $x^4 - 3x^2 - 4 = 0$

$$\frac{p}{q} = \frac{\pm 1 \ \pm 2 \ \pm 4}{+1} = \pm 1 \ \pm 2 \ \pm 4$$

Graph it! $\longrightarrow$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -3 & 0 & -4 \\ & \downarrow & 2 & 4 & 2 & 4 \\ \hline & 1 & 2 & 1 & 2 & \boxed{0} \end{array}$$

$x^3 + 2x^2 + x + 2$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 1 & 2 \\ & \downarrow & -2 & 0 & -2 \\ \hline & 1 & 0 & 1 & \boxed{0} \end{array}$$

$x^2 + 1 = 0$

25. $3x^3 + 4x^2 - 6x - 8 = 0$

$$\frac{p}{q} = \frac{\pm 1 \ \pm 8 \ \pm 2 \ \pm 4}{\pm 1 \ \pm 3} = \pm 1 \ \pm 8 \ \pm 2 \ \pm 4 \ \pm 1/3 \ \pm 8/3 \ \pm 2/3 \ \pm 4/3$$

Graph it! $\longrightarrow$

$$\begin{array}{r|rrrr} 4/3 & 3 & 4 & -6 & -8 \\ & \downarrow & 4 & 10\tfrac{2}{3} & 56/9 \\ \hline & 3 & 8 & 4\tfrac{2}{3} & \boxed{-16/9} \end{array}$$

NOT zero!

$$\begin{array}{r|rrrr} -4/3 & 3 & 4 & -6 & -8 \\ & \downarrow & -4 & 0 & 8 \\ \hline & 3 & 0 & -6 & \boxed{0} \end{array}$$

$3x^2 - 6 = 0$

$3x^2 = 6$

$x^2 = 2$

$$\boxed{x = \pm\sqrt{2} \qquad x = -4/3}$$

$\longrightarrow$

26. $x^4 - 2x^3 - 2x^2 = 0$

$x^2(x^2 - 2x - 2) = 0$

$$x = \frac{2 \pm \sqrt{4 - 4(1)(-2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4 + 8}}{2}$$

$$= \frac{2 \pm \sqrt{12}}{2}$$

$$= \frac{2 \pm \sqrt{4 \cdot 3}}{2}$$

$$= \frac{2 \pm 2\sqrt{3}}{2}$$

$$\boxed{x = 1 \pm \sqrt{3} \qquad x = 0 \text{ (twice)}}$$