☆ Review Synthetic with a factor given
$(x-a) \rightarrow a = \text{zero}$

$x^2 + 2x - 3 \; ; \; (x-1)$

Rational Root Theorem

- Given a polynomial that is NOT readily factorable
- How do we come up with solutions?

$\frac{p}{q}$ will give us possible roots

p = factors of the constant term
q = factors of the first term

Example: $x^3 + 10x^2 + 17x = 28$ Find solutions.

- Must be in standard form $\longrightarrow$ set = to 0
- Doesn't appear to be factorable

$$\frac{p}{q} = \frac{\pm 1 \;\; \pm 28 \;\; \pm 2 \;\; \pm 14 \;\; \pm 4 \;\; \pm 7}{\pm 1}$$

- These are all of the possible roots we could check with synthetic division.
- 1 looks like an easy root to check

$$\begin{array}{r|rrrr} 1 & 1 & 10 & 17 & -28 \\ & \downarrow & 1 & 11 & 28 \\ \hline & 1 & 11 & 28 & \boxed{0} \end{array}$$

$= x^2 + 11x + 28$

$(x+4)(x+7)$

- Can we factor this?
  YES!

$x+4=0$ $\quad$ $x+7=0$

$\boxed{x=1}$ $\quad$ $\boxed{x=-4}$ $\quad$ $\boxed{x=-7}$

- These are our 3 roots/solutions.
- Correlation between degree and # of zeros

Example Find all the real roots of
$4x^4 - 21x^3 + 18x^2 + 19x - 6$

- Use Rational Root Theorem

$\frac{p}{q} = \frac{\pm 1 \ \pm 2 \ \pm 3 \ \pm 6}{\pm 1 \ \pm 2 \ \pm 4} = \pm 1 \ \pm 2 \ \pm 3 \ \pm 6 \ \pm 1/2 \ \pm 3/2 \ \pm 1/4 \ \pm 3/4$

- Graph on calculator

$$\begin{array}{r|rrrrr} 2 & 4 & -21 & 18 & 19 & -6 \\ & \downarrow & 8 & -26 & -16 & 6 \\ \hline & 4 & -13 & -8 & 3 & \boxed{0} \end{array}$$

$= 4x^3 - 13x^2 - 8x + 3$ We still can't factor this.

Look at root between 3 and 4
Does it match up to anything? NO

Try 1/4 using synthetic. $\longrightarrow$ Doesn't work.
Just means 1/4 isn't a root.

$$\begin{array}{r|rrrr} -3/4 & 4 & -13 & -8 & 3 \\ & \downarrow & -3 & 12 & -3 \\ \hline & 4 & -16 & 4 & \boxed{0} \end{array}$$

$= 4x^2 - 16x + 4$

- Now we have a quadratic
- We can use quadratic formula.

- Solve out. $\longrightarrow$ Get 4 zeros.

$(2 \pm \sqrt{3})$ Matches our original degree.

Assignment: pg 442 # 11-14 # 24-26 } Adjust!
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