Mr. Ward Answer Key

Series

A **series** is the indicated sum of the terms of a sequence.

The following chart should give a good explanation of the difference between a sequence and a series.

Sequence	1, 2, 3, 4	2, 4, 6, 8, ...	$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$
Series	$1 + 2 + 3 + 4$	$2 + 4 + 6 + 8 + \cdots$	$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$

In your own words, write out the difference between a sequence and a series.

Sequence – a list of numbers

Series – summation of a sequence of numbers

Typically mathematicians will write series in what's called **Sigma notation**. We call it this because it uses the Greek letter Σ (capital *sigma*) to denote the sum of a sequence.

Example #1

Write the series 3 + 6 + 9 + 12 + 15 in Sigma notation.

Step 1: Try to find a rule/formula to describe the sequence. For example because I know we are going to start with the number 1, I'm asking myself, "what times 1 equals 3?"

n = 1

I then come up with this formula: $a_n = 3n$

Take a quick second to make sure that the formula we've come up with will match our sequence for all values of n.

$a_1 =$ 3 $a_2 =$ 6 $a_3 =$ 9 $a_4 =$ 12 $a_5 =$ 15

Now that we've confirmed that we've got the right formula we're going to write it out in **Sigma notation**.

Answer: $\sum_{n=1}^{5} 3n$

Look at the different parts for a second to understand where they all come from. First, look at the bottom part, "n=1". This is the first value for n. Consequently the "5" in the top stands for the last value of n. The "3n" is the formula we came up with.

Practice Problem #1

Write the series –2 + 4 – 6 + 8 – 10 + 12 in Sigma notation.

Before we begin, we're going to have to think about how to create our formula so that we alternate the negative sign each time. To do this you need to have a $(-1)^n$ in front of our formula. This will cause the sign to switch each time.

Now come up with rest of the formula.

What will the series look like in Sigma notation: $\sum_{n=1}^{6} (-1)^n 2n$

Example #2

Expand the series and evaluate. $\sum_{k=1}^{4}(2k-1)$

Now instead of creating the formula for Sigma notation, we're going to take the Sigma notation, write it out, and then find the sum of the series.

Step 1: Write out the series. (*Fill in the rest of the table on your own.*)

k	$2k-1$	a_k
1	2(1) – 1	1
2	2(2) – 1	3
3	2(3) – 1	5
4	2(4) – 1	7

The series then is 1 + 3 +5 + 7.

Step 2: Find the sum.

1 + 3 + 5 + 7 = 16

(Answer: 16)

You should now be able to say you can do the following things:

- I know the difference between a sequence and a series. (B1)
- I can expand a series from Sigma notation. (B2)
- I can write a series in Sigma notation. (B3)

Assignment: pg 874 #2-8, 13-19, 24-32 evens, 36-41

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pg 874

2. $\sum_{k=1}^{5} \frac{1}{k^2}$ 3. $\sum_{k=1}^{5} (-1)^k 3k$

4. $\sum_{k=1}^{5} 10^{k-1}$ 5. $\sum_{k=1}^{5} 100 - 5(k-1)$

6. $1 + 8 + 27 + 64 + 125 = 225$ 7. $12 - 3 + 4/3 - 3/4 =$
$\frac{144}{12} - \frac{36}{12} + \frac{16}{12} - \frac{9}{12} = \boxed{115/12}$

8. $-25 + -30 + -35 + -40 + -45 + -50 = -225$

13. $\sum_{k=1}^{5} 1.1k$ 14. $\sum_{k=1}^{5} \frac{k}{k+1}$ 15. $\sum_{k=1}^{6} (-1)^{k+1}(k+10)$

16. $\sum_{k=1}^{6} 2^{k-1}$ 17. $16 + 24 + 32 + 40 + 48 = 160$

18. $4 - 8 + 16 - 32 + 64 - 128 = -84$ 19. $0 + 1/3 + 2/4 + 3/5 =$
$0 + 1/3 + 1/2 + 3/5 =$
$0 + \frac{10}{30} + \frac{15}{30} + \frac{18}{30} = \boxed{43/30}$

24. $\sum_{k=1}^{6} (-1)^k k^2$ 26. $\sum_{k=1}^{5} \left(\frac{1}{3}\right)^k$ 28. $\sum_{k=1}^{4} 10.8 - 0.3(k-1)$

30. $\sum_{k=1}^{5} (-1)^k\ 3.4 + 0.5k$ 32. $\sum_{k=1}^{5} \frac{3}{k}$

36-41

36. $2 + 5 + 10 + 17 + 26 + 37 = 97$ 37. $-5 + 10 - 15 + 20 - 25 + 30 = 15$

38. $\frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \frac{1}{12} =$
$\frac{20}{120} + \frac{15}{120} + \frac{12}{120} + \frac{10}{120} = \frac{57}{120} = \boxed{\frac{19}{40}}$ 39. $1 + 4 + 7 + 10 + 13 + 16 = 51$

40. $48 + 60 + 72 + 84 + 96 + 108 = \boxed{468}$ 41. $\frac{1}{5} + \frac{4}{10} + \frac{9}{15} + \frac{16}{20} + 1$
$\frac{12}{60} + \frac{24}{60} + \frac{36}{60} + \frac{48}{60} + \frac{60}{60} = \frac{180}{60} = \boxed{3}$