

## Exponentials and Logarithms

A logarithmic function is the inverse of an exponential function. More specifically a logarithm is the exponent to which a specified base is raised. To better understand this let's look at the two equations below and see how they relate to each other.

Exponential Equation	Logarithmic Equation
$b^x = a$	$\log_b a = x$

Look carefully at where the "a", "b", and the "x" all move to when we switch between exponents and logarithms. **You need to be good at doing this switching!**

Example #1 Write the exponential equation in logarithmic form.

$$4^3 = 64$$

First thing we do is identify which numbers are a, b, and x. Then we simply convert it over to logarithmic form.

What is a? 64 What is b? 4 What is x? 3

Write out your answer in logarithmic form.  $\log_4 64 = 3$

You should have gotten  $\log_4 64 = 3$ .

Example #2 Write the logarithmic equation in exponential form.

$$\log_6 36 = 2$$

Again identify which numbers are a, b, and x.

What is a? 36 What is b? 6 What is x? 2

Write out your answer in exponential form.  $6^2 = 36$

Hopefully you got  $6^2 = 36$ . If not raise your hand and get some help!

**You need to get to the point where you can switch between exponents and logarithms in your head, WITHOUT identifying what "a", "b", and "x" are.**

*Special Case:* Logarithm of 1

$$\log_b 1 = 0 \rightarrow b^0 = 1 \text{ (Remember, anything to the 0 power is 1.)}$$

## Graphing Logarithms

Calculators can't graph logarithmic functions. We get around this by graphing their inverses, exponential functions (*This is why we did the things we've done the past two days!!*). To do this we look at the table of values of a logarithmic function and just flip the table of values. Here's what a question like this will look like.

**Example #1** Use the given x-values to graph the function  $f(x) = 3^x$ ;  $x = -2, -1, 0, 1, 2$ . Then graph it's inverse.

**Step 1:** Create a table of values using the given x-values.

**Step 2:** Plug in the different x-values and solve for the y-values. **Then graph!**

Original

x	y
-2	$1/9$
-1	$1/3$
0	1
1	3
2	9

**Step 3:** Switch the x and y coordinates. (This gives us the inverse. Remember?!)

Inverse

x	y
$1/9$	-2
$1/3$	-1
1	0
3	1
9	2

**Step 4:** Graph the logarithmic function (which is the inverse of the exponential function) using the new table of values. You should now have two different functions on your graph paper.

**Assignment:** Practice B WS + pg 509 #2-15

# Mr. Ward Answer Key

pg 509

2.  $\log_{2.4} 1 = 0$     3.  $\log_4 8 = 1.5$     4.  $\log_{10} 0.01 = -2$

5.  $\log_3 243 = x$     6.  $4^{-2} = 0.0625$     7.  $x^3 = -16$

8.  $0.9^2 = 0.81$     9.  $6^3 = x$     10.  $7^x = 243$   
 $= 3$

11.  $3^x = \frac{1}{9}$     12.  $0.5^x = 0.25$     13.  $1.2^x = 1.44$   
 $= -2$                        $= 2$                        $= 2$

14.	Original	Inverse	15.	Original	Inverse																																																
	<table border="1"><tr><td>x</td><td>y</td></tr><tr><td>-2</td><td>1/25</td></tr><tr><td>-1</td><td>1/5</td></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>5</td></tr><tr><td>1.5</td><td>11.2</td></tr></table>	x	y	-2	1/25	-1	1/5	0	1	1	5	1.5	11.2	<table border="1"><tr><td>x</td><td>y</td></tr><tr><td>1/25</td><td>-2</td></tr><tr><td>1/5</td><td>-1</td></tr><tr><td>1</td><td>0</td></tr><tr><td>5</td><td>1</td></tr><tr><td>11.2</td><td>1.5</td></tr></table>	x	y	1/25	-2	1/5	-1	1	0	5	1	11.2	1.5		<table border="1"><tr><td>x</td><td>y</td></tr><tr><td>-2</td><td>4</td></tr><tr><td>-1</td><td>2</td></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>1/2</td></tr><tr><td>2</td><td>1/4</td></tr></table>	x	y	-2	4	-1	2	0	1	1	1/2	2	1/4	<table border="1"><tr><td>x</td><td>y</td></tr><tr><td>4</td><td>-2</td></tr><tr><td>2</td><td>-1</td></tr><tr><td>1</td><td>0</td></tr><tr><td>1/2</td><td>1</td></tr><tr><td>1/4</td><td>2</td></tr></table>	x	y	4	-2	2	-1	1	0	1/2	1	1/4	2
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See Graph

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**LESSON** **Practice B**  
**7-3** **Logarithmic Functions**

Write each exponential equation in logarithmic form.

1.  $3^7 = 2187$

$\log_3 2187 = 7$

2.  $12^2 = 144$

$\log_{12} 144 = 2$

3.  $5^3 = 125$

$\log_5 125 = 3$

Write each logarithmic equation in exponential form.

4.  $\log_{10} 100,000 = 5$

$10^5 = 100,000$

5.  $\log_4 1024 = 5$

$4^5 = 1024$

6.  $\log_9 729 = 3$

$9^3 = 729$

Evaluate by using mental math.

7.  $\log 1,000,000$   $10^x = 1,000,000$

$= 6$

8.  $\log 10$

$= 1$

9.  $\log 1$

$= 0$

10.  $\log_4 16$

$= 2$

11.  $\log_8 1$

$= 0$

12.  $\log_5 625$   $5^x = 625$

$= 4$

Use the given x-values to graph each function. Then graph its inverse. Describe the domain and range of the inverse function.

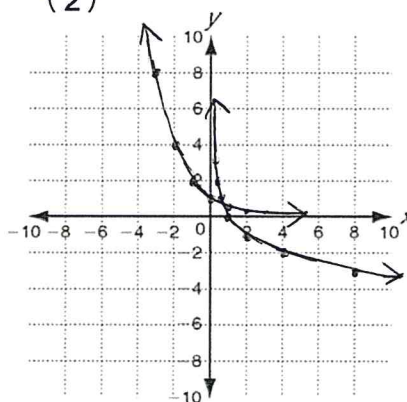
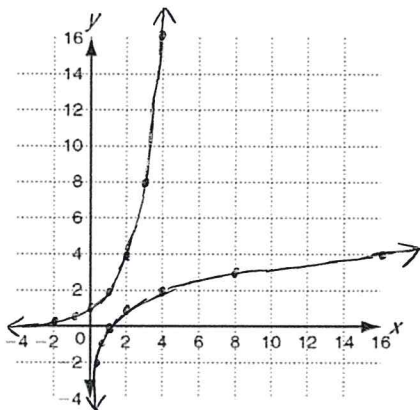
13.  $f(x) = 2^x$ ;  $x = -2, -1, 0, 1, 2, 3, 4$

14.  $f(x) = \left(\frac{1}{2}\right)^x$ ;  $x = -3, -2, -1, 0, 1, 2, 3$

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8
4	16

⇒

x	y
1/4	-2
1/2	-1
1	0
2	1
4	2
8	3
16	4



x	y
-3	8
-2	4
-1	2
0	1
1	1/2
2	1/4
3	1/8

⇒

x	y
8	-3
4	-2
2	-1
1	0
1/2	1
1/4	2
1/8	3

**Solve.**

15. The hydrogen ion concentration in moles per liter for a certain brand of tomato-vegetable juice is 0.000316.

- Write a logarithmic equation for the pH of the juice.
- What is the pH of the juice?

