Name *Mr. Ward* Date ________ Class ________

LESSON 14-4

Reteach

Sum and Difference Identities

You can use angle addition and subtraction identities to find the exact value of some trigonometric expressions.

Look for ways to use 30° or $\frac{\pi}{6}$, 45° or $\frac{\pi}{4}$, and 60° or $\frac{\pi}{3}$ to make the sum because exact values are known for these angles.

Sum Identities	**Difference Identities**
$\sin(A+B) = \sin A\cos B + \cos A\sin B$	$\sin(A-B) = \sin A\cos B - \cos A\sin B$
$\cos(A+B) = \cos A\cos B - \sin A\sin B$	$\cos(A-B) = \cos A\cos B + \sin A\sin B$
$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$	$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

Find the exact value of cos105°.

$\cos 105° = \cos(60° + 45°)$ *Think: 60° + 45° = 105°.*

$= \cos 60°\cos 45° - \sin 60°\sin 45°$ *Use cos(A + B) identity.*

Substitute: $A = 60°$ and $B = 45°$.

$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$ *Evaluate.*

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4}$ *Simplify.*

The value is negative. This makes sense since 105° lies in Quadrant II where cosine is negative.

Find the exact value of $\sin\left(\frac{\pi}{12}\right)$.

$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$ *Think: $\frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$.*

$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$ *Use sin(A – B) identity.*

Substitute: $A = \frac{\pi}{3}$ and $B = \frac{\pi}{4}$.

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$ *Evaluate.*

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$ *Simplify.*

Use the sum or difference identity to find the exact value of each expression.

1. $\cos(-15°) = \cos(30° - 45°)$

$\cos 30 \cos 45 + \sin 30 \sin 45$

$\sqrt{3}/2 \cdot \sqrt{2}/2 + 1/2 \cdot \sqrt{2}/2$

$\sqrt{6}/4 + \sqrt{2}/4$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$

2. $\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

105° 60° 45°

$\sin 60 \cos 45 + \cos 60 \sin 45$

$\sqrt{3}/2 \cdot \sqrt{2}/2 + 1/2 \cdot \sqrt{2}/2$

$\sqrt{6}/4 + \sqrt{2}/4$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$

Name ______________________ Date ______________ Class ______________

LESSON 14-4

Reteach

Sum and Difference Identities *(continued)*

You can use angle addition and subtraction identities to prove identities.
Use an identity to make one side of the equation resemble the other side.

Sum Identities	Difference Identities
$\sin(A+B) = \sin A\cos B + \cos A\sin B$	$\sin(A-B) = \sin A\cos B - \cos A\sin B$
$\cos(A+B) = \cos A\cos B - \sin A\sin B$	$\cos(A-B) = \cos A\cos B + \sin A\sin B$
$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$	$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

Prove: $\tan(\pi + x) = \tan x$

$\tan(\pi + x) = \tan x$ — *Modify the left side.*

$\dfrac{\tan\pi + \tan x}{1 - \tan\pi\tan x} = \tan x$ — *Use tan(A + B) identity.*

Substitute: $A = \pi$ and $B = x$.

$\dfrac{0 + \tan x}{1 - (0)\tan x} = \tan x$ — *Evaluate. Think: $\tan\pi = 0$.*

$\dfrac{\tan x}{1} = \tan x$ — *Simplify.*

Prove: $\sin(\pi - x) = \sin x$

$\sin(\pi - x) = \sin x$ — *Modify the left side.*

$\sin\pi\cos x - \cos\pi\sin x = \sin x$ — *Use sin(A – B) identity.*

$(0)\cos x - (-1)\sin x = \sin x$ — *Evaluate. Think: $\sin\pi = 0$ and $\cos\pi = -1$.*

$0 + \sin x = \sin x$ — *Simplify.*

Write the missing steps or reasons to prove each identity.

3. $\sin\left(\dfrac{\pi}{2} + x\right) = \cos x$ — Modify the left side.

$\sin\left(\dfrac{\pi}{2}\right)\cos x + \cos\left(\dfrac{\pi}{2}\right)\sin x = \cos x$ — Use sin (A + B) identity.

$1 \cdot \cos x + 0 \cdot \cancel{\sin x} = \cos x$ — Evaluate.

$\cos x = \cos x$ — Simplify.

4. $\cos(\pi + x) = -\cos x$ — Modify the left side.

$\cos\pi\cos x - \sin\pi\sin x = -\cos x$ — Use cos(A + B) identity.

$-1 \cdot \cos x - 0 \cdot \cancel{\sin x} = -\cos x$ — Evaluate

$-\cos x = -\cos x$ — Simplify

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LESSON 14-4

Practice B

Sum and Difference Identities

Find the exact value of each expression.

1. $\cos 120°$ (On UC)

   $-1/2$

2. $\sin 315°$ (On UC)

   $-\sqrt{2}/2$

3. $\tan 255°$

   See Attached

4. $\tan\frac{7\pi}{6}$ (210°) (On UC)

   $\frac{-1/2}{-\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \boxed{\sqrt{3}/3}$

5. $\sin\frac{\pi}{12}$ (15°)

   See Attached

6. $\cos\frac{3\pi}{4}$ (135°) (On UC)

   $-\sqrt{2}/2$

Prove each identity.

7. $\sin\left(x - \frac{3\pi}{2}\right) = \cos x$

   $\sin x \cos\frac{3\pi}{2} - \cos x \sin\frac{3\pi}{2} = \cos x$

   $\cancel{\sin x \cdot 0} - \cos x \cdot -1 = \cos x$

   $-\cos x \cdot -1 = \cos x$

   $\cos x = \cos x$

8. $\cos\left(x - \frac{\pi}{2}\right) = \sin x$

   $\cos x \cos\frac{\pi}{2} + \sin x \sin\frac{\pi}{2} = \sin x$

   $\cancel{\cos x \cdot 0} + \sin x \cdot 1 = \sin x$

   $\sin x = \sin x$

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LESSON 14-4

Practice C

Sum and Difference Identities

Find the exact value of each expression.

1. $\cos 300°$ (On UC)

   $1/2$

2. $\sin(-15°)$

   See Attached

3. $\tan 285°$

   See Attached

4. $\tan\frac{11\pi}{12}$ (165°)

   See Attached

5. $\sin\frac{13\pi}{12}$ (195°)

   See Attached

6. $\cos\left(-\frac{13\pi}{12}\right)$ (−195°)

   See Attached

Prove each identity.

7. $\tan(\pi - x) = -\tan x$

   $\frac{\tan\pi - \tan x}{1 + \tan\pi \tan x} = -\tan x$

   $\frac{0 - \tan x}{1 + (\cancel{0 \cdot \tan x})} = -\tan x$

   $-\tan x = -\tan x$

8. $\sin\left(\frac{\pi}{2} - x\right) = \cos x$

   $\sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2} \cdot \sin x = \cos x$

   $1 \cdot \cos x - \cancel{0 \cdot \sin x} = \cos x$

   $\cos x = \cos x$

Mr. Ward Answer Key

Practice B

3. $\tan 255 = \tan(225 + 30)$

$$= \frac{\tan 225 + \tan 30}{1 - (\tan 225 \cdot \tan 30)}$$

$$= \frac{1 + \sqrt{3}/3}{1 - (1 \cdot \sqrt{3}/3)}$$

$$= \boxed{\frac{1 + \sqrt{3}/3}{1 - \sqrt{3}/3}}$$

5. $\sin 15° = \sin(45° - 30°)$

$$= \sin 45 \cos 30 - \cos 45 \sin 30$$

$$= \sqrt{2}/2 \cdot \sqrt{3}/2 - \sqrt{2}/2 \cdot 1/2$$

$$= \sqrt{6}/4 - \sqrt{2}/4$$

$$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

Practice C

2. $\sin(-15°) = \sin(30° - 45°)$

$$= \sin 30 \cos 45 - \cos 30 \sin 45$$

$$= 1/2 \cdot \sqrt{2}/2 - \sqrt{3}/2 \cdot \sqrt{2}/2$$

$$= \sqrt{2}/4 - \sqrt{6}/4$$

$$= \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$$

3. $\tan 285 = \tan(240° + 45°)$

$$= \frac{\tan 240 + \tan 45}{1 - (\tan 240 \cdot \tan 45)}$$

$$= \frac{\sqrt{3} + 1}{1 - (\sqrt{3} \cdot 1)}$$

$$= \boxed{\frac{\sqrt{3} + 1}{1 - \sqrt{3}}}$$

4. $\tan(165°) = \tan(135° + 30°)$

$$= \frac{\tan 135 + \tan 30}{1 - (\tan 135 \cdot \tan 30)}$$

$$= \frac{-1 + \sqrt{3}/3}{1 - (-1 \cdot \sqrt{3}/3)}$$

$$= \boxed{\frac{-1 + \sqrt{3}/3}{1 + \sqrt{3}/3}}$$

5. $\sin(195°) = \sin(150° + 45°)$

$$= \sin 150 \cos 45 + \cos 150 \sin 45$$

$$= 1/2 \cdot \sqrt{2}/2 + \sqrt{3}/2 \cdot \sqrt{2}/2$$

$$= \sqrt{2}/4 + \sqrt{6}/4$$

$$= \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}}$$

6. $\cos(-195°) = \cos(45 - 240)$

$$= \cos 45 \cos 240 + \sin 45 \sin 240$$

$$= \sqrt{2}/2 \cdot -1/2 + \sqrt{2}/2 \cdot -\sqrt{3}/2$$

$$= -\sqrt{2}/4 + -\sqrt{6}/4$$

$$= \boxed{\frac{-\sqrt{2} - \sqrt{6}}{4}}$$