Mr. Ward Answer Key

Variance and Standard Deviation

Variance (σ^2) – the average of the squared differences from the mean

Standard Deviation (σ) – the square root of the variance

- Low standard deviations indicate data that are clustered near the measures of central tendency (mean, median, mode).
- High standard deviations indicate data that are spread out from the center.

Practice Problem #1

The following data represents the number of free throws Mike made after 10 sets of 50 free throws.

35, 40, 38, 43, 38, 36, 46, 47, 40, 37

Find the mean and standard deviation of the data.

Data Value x	35	40	38	43	38	36	46	47	40	37
$x - \bar{x}$	-5	0	-2	3	-2	-4	6	7	0	-3
$(x - \bar{x})^2$	25	0	4	9	4	16	36	49	0	9

1. What is the mean? 40
This value will be represented by $\bar{x}$.

2. Now take each data value x and subtract the mean from it. Fill in the 2nd row with your answers.

3. Now for the third row, take your answers from the 2nd row and square them.

To find the variance we are going to add up every number in the third row and divide by the total number of trials.

$$\text{Variance} = \frac{25 + 0 + 4 + 9 + 4 + 16 + 36 + 49 + 0 + 9}{10} = 15.2$$

Now to find the standard deviation we need to take the square root of the variance.

$$\text{Standard Deviation} = \sqrt{15.2} = 3.9$$

** This means that most of the data is within ± 3.9 of the mean 40. ** Let's look at another set of data to see if we can't start to get a better understanding about standard deviation.

Practice Problem #2

The following data represents the number of free throws Austin made after 10 sets of 50 free throws.

23, 46, 34, 12, 8, 48, 29, 40, 6, 44

Find the mean and standard deviation of the data.

Data Value x	23	46	34	12	8	48	29	40	6	44
$x - \bar{x}$	-6	17	5	-17	-21	19	0	11	-23	15
$(x - \bar{x})^2$	36	289	25	289	441	361	0	121	529	225

Mean = 29

Variance = 231.6

Standard Deviation = 15.2

What does the difference in standard deviation from Mike's data set to Austin's data set mean? Jot some thoughts down.

Mike is a much more consistent free throw shooter. His data is clustered much closer together than Austin's data.

Outliers

Outliers are extreme values that are either much greater or much less than the other data values. Because of this they have a huge effect on the mean and standard deviation. Therefore in many experiments and data sets the outlier is typically removed. But how do we decide what constitutes an outlier? One way is to look at data values that are more than 3 standard deviations away from the mean.

For example, let's look at this scenario. *mean* = 41.2 and *standard deviation* = 3.6

Since we want values that are 3 standard deviations away, 3(3.6) = 10.8

Now both subtract and add 10.8 to the mean of 41.2 to find this range.

Any value greater than 52 and any value less than 30.4 would be considered an outlier in the data set.

Assignment: Problem Solving Worksheet + Practice B Worksheet

Name Mr. Ward Date _____ Class _____

LESSON 11-5

Problem Solving

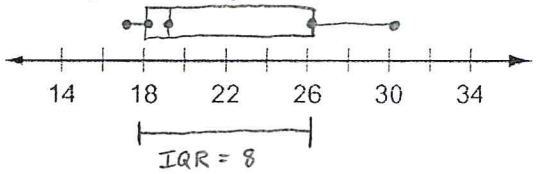
Measures of Central Tendency and Variation

Each week, Damien records the miles per gallon for his car, to the nearest whole number. Over a period of 10 weeks, the data are 18, 17, 19, 18, 18, 25, 29, 30, 26, 19. He wants to arrange and summarize his data so that he can analyze it.

1. Make a box-and-whisker plot of his data.
   a. Order the data from least to greatest. 17, 18, 18, 18, 19, 19, 25, 26, 29, 30
   b. Identify the minimum, maximum, median, first quartile, and third quartile.

   min = 17 max = 30 meadian = 19 Q1 = 18 Q3 = 26

   c. Use the number line to make a box-and-whisker plot of the data. Find and label the interquartile range.



   d. Explain what the interquartile range represents in terms of the car's miles per gallon.

   50% of the time the car will get between 18-26 mpg

2. Find the standard deviation for the data.
   a. Write an equation and solve to find the mean. $\frac{18+17+19+18+18+25+29+30+26+19}{10} = 21.9$
   b. Complete the table to show the difference between the mean and each data value, and the square of that difference.

Data Value, *x*	18	17	19	18	18	25	29	30	26	19
$x - \bar{x}$	-3.9	-4.9	-2.9	-3.9	-3.9	3.1	7.1	8.1	4.1	-2.9
$(x - \bar{x})^2$	15.21	24.01	8.41	15.21	15.21	9.61	50.41	65.61	16.81	8.41

   c. Explain how to use the data from the table to find the standard deviation.

   Take the square root of the mean of all the values in row 3

Variance = 22.89

   d. What is the standard deviation for the data? st. dev = 4.78
   e. Explain what the standard deviation represents in terms of the car's miles per gallon.

   the car's mpg will for the most part only fluctuate ± 4.78 mpg

3. Damien thinks that the standard deviation is a more reliable measure of dispersion than the interquartile range. Is he correct? Explain.

   He is correct. The standard deviation takes all values into consideration where as the IQR is only determined by the values Q1 and Q3.

Name Mr. Ward Date _____ Class _____

LESSON 11-5

Practice B

Measures of Central Tendency and Variation

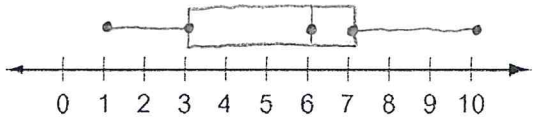
Find the mean, median, and mode of each data set.

1. { 12, 11, 17, 3, 9, 14, 16, 2 } (2, 3, 9, 11, 12, 14, 16, 17)
   a. Mean 10.5
   b. Median 11.5
   c. Mode N/A

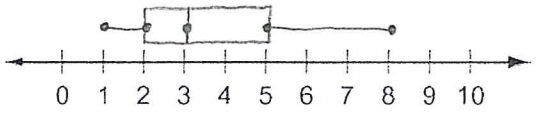
2. { 6, 9, 9, 20, 4, 5, 9, 13, 10, 1 } (1, 4, 5, 6, 9, 9, 9, 10, 13, 20)
   a. Mean 8.6
   b. Median 9
   c. Mode 9

Make a box-and-whisker plot of the data. Find the interquartile range.

3. { 3, 7, 7, 3, 10, 1, 6, 6 } (1, 3, 3, 6, 6, 7, 7, 10)
   IQR = 4



4. { 1, 2, 3, 5, 3, 5, 8, 2 } (1, 2, 2, 3, 3, 5, 5, 8)
   IQR = 3



Find the variance and standard deviation.

See separate sheet

5. { 7, 4, 3, 9, 2 }

6. { 35, 67, 21, 16, 24, 51, 18, 32 }

7. { 19, 23, 17, 20, 25, 19, 15, 22 }

8. { 5, 12, 10, 13, 8, 11, 15, 12 }

Solve.

9. The probability distribution for the amount of rain that falls on Boston in May each year is given below. Find the expected amount of rain for Boston in May.

Inches of Rain, *n*	5	6	7	8
Probability	0.05	0.10	0.64	0.21

10. A biologist is growing bacteria in the lab. For a certain species of bacteria, she records these doubling times: 41 min, 45 min, 39 min, 42 min, 38 min, 88 min, 43 min, 40 min, 44 min, 39 min, 42 min, and 40 min.
    a. Find the mean of the data. 45.1
    b. Find the standard deviation. 13.1
    c. Identify any outliers. 88 min is outlier $45.1 + 3(13.1) = 84.8$ $84.8 < 88$
    d. Describe how any outlier affects the mean and the standard deviation. increase mean and dramatically increases st. deviation

x	41	45	39	42	38	88	43	40	44	39	42	40
$x-\bar{x}$	-4.1	-0.1	-6.1	-3.1	-7.1	42.9	-2.1	-5.1	-1.1	-6.1	-3.1	-5.1
$(x-\bar{x})^2$	16.81	.01	37.21	9.61	50.41	1840.41	4.41	26.01	1.21	37.21	9.61	26.01

variance = 171.58

Practice B WS

5. mean = 5

x	7	4	3	9	2
$x-\bar{x}$	2	-1	-2	4	-3
$(x-\bar{x})^2$	4	1	4	16	9

variance = $\frac{4+1+4+16+9}{5}$ = 6.8

st. dev = $\sqrt{6.8}$ = 2.61

6. mean = 33

x	35	67	21	16	24	51	18	32
$x-\bar{x}$	2	34	-12	-17	-9	18	-15	-1
$(x-\bar{x})^2$	4	1156	144	289	81	324	225	1

variance = 278 st. dev = 16.67

7. mean = 20

x	19	23	17	20	25	19	15	22
$x-\bar{x}$	-1	3	-3	0	5	-1	-5	2
$(x-\bar{x})^2$	1	9	9	0	25	1	25	4

variance = 9.25 st. dev = 3.04

8. mean = 10.75

x	5	12	10	13	8	11	15	12
$x-\bar{x}$	-5.75	1.25	-0.75	2.25	-2.75	.25	4.25	1.25
$(x-\bar{x})^2$	33.0625	1.5625	.5625	5.0625	7.5625	.0625	18.0625	1.5625

variance = 8.44 st. dev = 2.9